THEORY OF THE CARTOGRAPHIC LINE REVISITED/ IMPLICATIONS FOR AUTOMATED GENERALIZATION

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ABSTRACT 'The Theory of a Cartographic Line' (Peucker 1975) describes width as being the essential characteristic of a cartographic line. Digital representations have tended to ignore this basic attribute and in the context of generalization the oversight is detrimental. The theory claims that a set of enclosing bands captures the cartographic character of width and supports generalization. The Douglas algorithm, still one of the most commonly used algorithms for generalizing digital representations, uses this model. Work of a Polish mathematician Perkal, provides the basis for another model of cartographic line width and a different generalization technique. This paper examines how effectively both models capture cartographic line width and succeed in producing generalized results, particularly for larger scale reductions. The two techniques are assessed by their ability to satisfy two objectives: capturing the essential and recognizable characteristics of geographic features and creating representations which can be legibly displayed at smaller scales. The paper compares the behavior of the two methods as applied to digital coastline data.

1. INTRODUCTION

Typically, an objective of cartographic generalization is to remove detail while retaining important information content and recognizable characteristics of the geography being represented (Pannekoek 1962, Tobler 1964, Jenks 1981, Imhof 1982). Another primary objective is to assure a legible representation (Pannekoek 1962, Robinson 1984, Keates 1989, Muller 1989). Both objectives must be satisfied for effective results. The challenge for automation thus becomes to define 'important' information or recognizable character, to assure legibility by locating and resolving spatial conflicts, and to relate these measures to desired scales. In the ideal case it should be possible to specify a target scale and have an automated process produce the appropriate generalization.

A significant amount of research in automated generalization has been devoted to techniques for capturing essential character or avoiding spatial conflicts (Mackaness 1987, Buttenfield 1989, Monmonier 1989, Muller 1990), but satisfaction of both objectives has not generally been used as performance criteria. Mathematical measures have been applied more frequently (McMaster 1983, McMaster 1987), but as Visvalingam and Whyatt (1990) point out, these can be inappropriate and misleading. This paper begins with a discussion of these generalization objectives and the ability to relate them to changes in scale.

1.1 *Defining Importance*

Humans have no difficulty identifying important information content, but transferring this skill to the computer is no easy task. The importance of cartographic features varies with the purpose of a map and is often a complex function of spatial context, thematic significance, or some combination of these. For example, a city may be important based on its large population, on its strategic location at the mouth of a river, or other more subjective criteria.

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Generalization algorithms, to date, have not developed sophisticated definitions of importance, but rely primarily on geometric criteria (Zoraster 1984, McMaster 1987). Geometric criteria which have been used as measures of importance include size, shape, inflection points, and intersections (Pavlidis 1978). With respect to line generalization, the identification of critical points (end points, inflection points, local maxima and minima) has received the most attention (Freeman 1978, Marino 1979, Dettori 1982, Thapa 1988). While geometric criteria alone cannot provide a complete definition of importance they play a useful role.

Often the importance of geographic features is scale dependent, thus instructions to the computer to identify and retain important information should be sensitive to the degree of scale change. In using geometric criteria, we should consider whether the definition of importance is valid for a broad range of scales or applicable only to a specific scale range. For example, a geometric measure of importance may generate effective results for a scale change of 1:24,000 to 1:50,000 but produce unacceptable results for a scale change from 1:24,000 to 1:250,000.

1.2 *Maintaining Legibility*

Maps represent geographic information through the construction of physical marks on paper. It is clear that as scale is decreased the total space available for these marks decreases, and the same size marks collide and overlap at smaller scales. This phenomenon requires that information (marks) be removed or modified to remain legible. Legibility requirements have been well documented by cartographers and map product specifications. These typically include the minimum dimensions of marks and minimum spacings between them. In contrast to importance, the relationship between scale and legibility is well known or at least predictable. For a given target scale and projected viewing distance, we can compute the minimum size, length, and spacing between objects required to clearly display them. The minimum size and length thresholds are straight forward to apply, for example, a filter which removes features below a specified size threshold. The challenge, however, is to locate where minimum spacing thresholds have been violated and apply corrective actions.

In the manual process of generalization, the physical width of a line symbol provides direct visual evidence of potential conflicts so a cartographer can see where modifications need to be made. Indeed a common practice in manual generalization is to use a wide pen to draft a generalized line. The wide pen is required for photo reductions but it also helps to visualize the potential collision of symbols that may occur for a particular scale reduction. The width of a cartographic line thus plays an important role in generalization. Indeed, the minimum spacing threshold between objects must account for the width of the graphic symbol (see Figure 1). This factor is obvious in a graphic medium, but has tended to be overlooked in the electronic environment.

The next section discusses computer representations of cartographic lines and cartographic line width in particular. Subsequent sections discuss the two models of cartographic lines and associated generalization processes. Both models are vector representations in which lines or boundaries of areas are modeled by

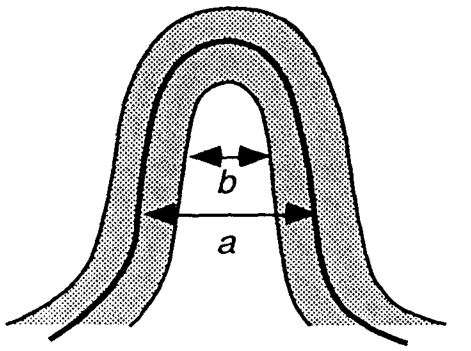


FIGURE 1. The minimum separation between marks must account for the symbolized width (*b*) rather than the distance between centroids or centerlines of marks (*a*).

straight line segments between points. The final section reports on the application of the two generalization methods to three geographically different coastline data sets. The generalized results produced by each method are then compared to results produced by traditional manual generalization.

2. BACKGROUND

2.1 *Computer Representations of Cartographic Lines*

The traditional mathematical representation of a line is as a locus of zero width. Although there are numerous mathematical expressions for a line, one way to express such a line is as:

L = an infinite set of points where points are represented by an ordered pair of coordinates

The typical vector representation of a cartographic line described in this paper approximates this mathematical concept of a line by a finite number of points and straight lines connecting them. True to the mathematical concept, this approximation maintains zero width.

Manipulations of vector models in a geographic information system generally do not take the symbolized widths of lines into account. When the operation is generalization and the product is a reduced scale display, the oversight becomes critical. In fact, when symbolization is considered only after generalization, results are often disappointing. Analysis and display operations could both benefit from incorporation of some explicit expression of width that captures the behavior of a cartographic line. A raster representation is one alternative as it expresses all objects as having some width. A raster model, however, cannot represent a line as having constant width. The width varies depending on cell shape, size, and orientation of a line with respect to cell orientation. The alternative to the raster model is a vector model which expresses the cartographic characteristic of width. Two types of bands have been used to approximate width in vector models.

2.1 *Expressing Width by Rectangular Bands*

Poiker, in this Theory of a Cartographic Line (Peucker 1975) emphasized the

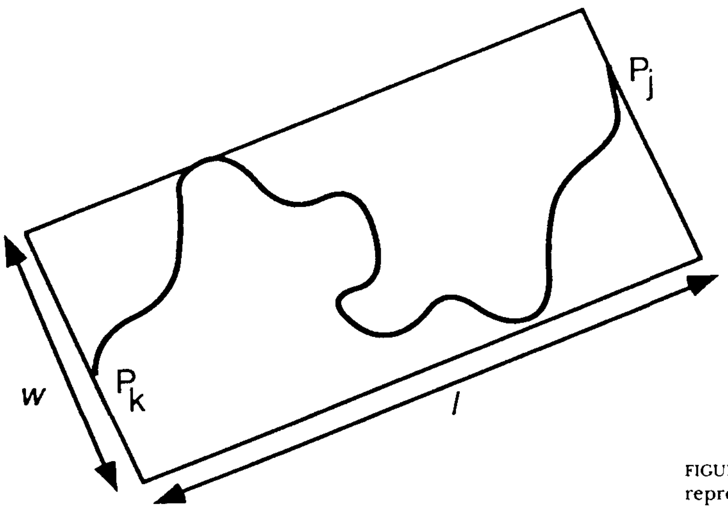


FIGURE 2. A cartographic line represented by a bounding rectangular band.

essential characteristic of the cartographic line as "having always a certain thickness". He proposed that a line of any extent can be defined by a general direction and a band with a width and a length (See Figure 2). The width of the band is assumed to approximate the width of the line.

In the Poiker model, as a line is decomposed into sub-component line segments, corresponding bands can be constructed in which the width of each subband approximates the width of the enclosed line segment. For a line L with n points ($L = p_i{}'1 < \; i < n$) the width of the band, w, is a function of two points, $w(p_k{}'p_j)$ and is a minimum real value such that all points in the part of the line between k and j are within the band. Because width is a function of different point combinations it is not constant. The set of widths includes $w_m(p_i{}'p_j)$, $1 \leq m \leq n - 1$ with values ranging from zero to the width of the band enclosing the entire line. Ballard (1981) expanded on this concept with the introduction of strip trees to represent lines as having locally varying thickness.

Poiker (Peucker 1975) suggested that the enclosing bands can represent different levels of abstraction of the line. The single band enclosing the entire line provides the highest level of abstraction. As the line is iteratively decomposed, the sets of resulting sub-bands provide increasingly more detailed representations of the line. The progression of bands can therefore reflect the cartographic phenomena of increasing width and greater abstraction associated with more generalized or smaller scale representations (See Figure 3).

These bands, however, deviate from symbolized line width in two ways. One difference is that the width of a cartographic line symbol is constant over its length (at least within the capability of the pen or print medium to maintain constant width). The other difference is that the approximating bands at the limit reduce to zero width while a graphic line always maintains some width.

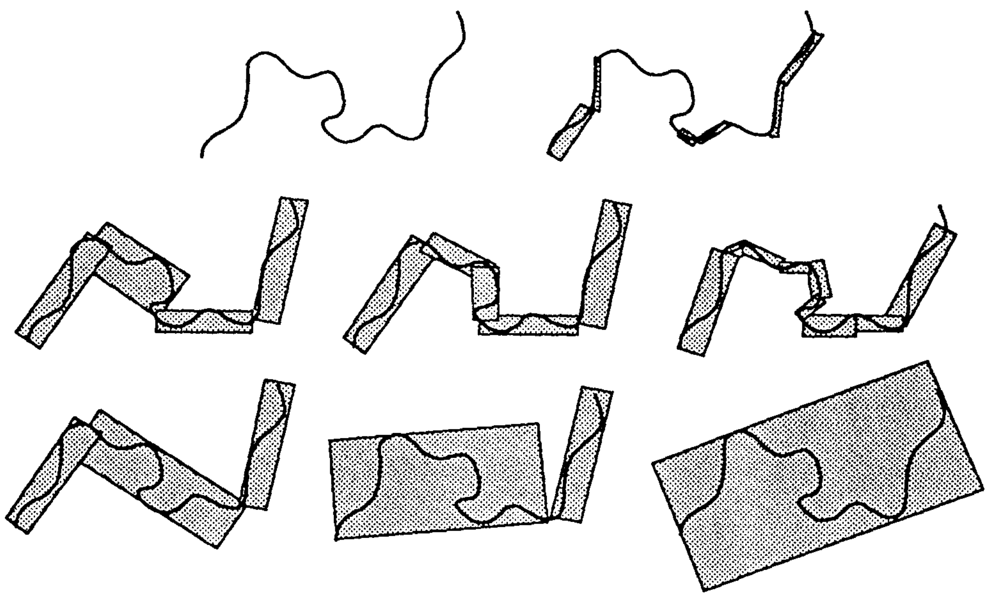


FIGURE 3. The progression of bands going from zero width to the widest, most abstract representation of the line. The band reduces to zero width for two point line segments.

2.2 *Generalization Based on Enclosing Bands*

The Douglas (Douglas and Peucker 1973) algorithm utilizes the band approximation as a basis for generalization of a line. The algorithm begins with construction of a trend line linking start and end points ($p_i'p_j$ in Figure 4). For points along the line, the vertical distance to the trend line is computed and the point with maximum absolute deviation from the trend line is retained (p_m in Figure 4). This point becomes a new anchor point for the trend line and the process continues until the maximum absolute distance no longer exceeds a pre-set threshold or tolerance. The points of maximum deviation and the anchor points dimension the band and the process iterates until the width of all sub-bands is less than or equal to the tolerance. The selected points (maximum deviation points) provide a new representation of the line, with all intermediate points being eliminated.

Points which are retained in this process have been described as critical points, similar to points a cartographer would select to represent a line (Marino 1979, White 1985, Jenks 1989). In other words, the selected points are those deemed to be important for maintaining the essential or recognizable character of the line. In this case, importance or essential character has been geometrically defined as a pre-set distance from a trend line. The validity of this criteria as a measure of importance was empirically established by both Marino (1979) and White (1985). We could therefore state that the algorithm meets the generalization objective of retaining important information. White's and Marino's studies, however, only tested for importance in the context of individual line segments. The Douglas algorithm, while it effectively defines important features within a line, may not successfully define them for a region. In the context of a map sheet or larger geographic area,

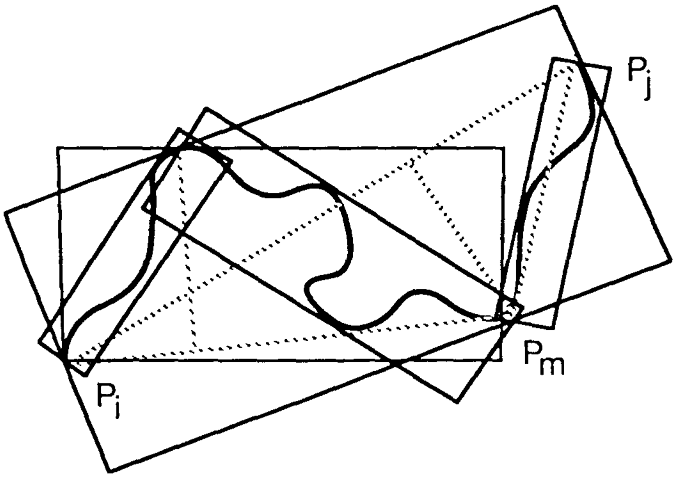


FIGURE 4. Generalization is accomplished by first constructing a trend line between end points pi and pj. Distances to the trend line are computed from points along the line and the point with the maximum absolute deviation (Pm) from the line is retained. This point becomes a new anchor point and the process iterates for each new segment until the maximum deviation no longer exceeds the tolerance.

the inflection or critical points within a line are often not the important information. Thapa (1988) and Monmonier (1986) made similar findings.

Maintenance of legibility is not a concern addressed by this algorithm. The band-width is not used as a cartographer uses symbol width to detect spatial collisions of a line with itself or other nearby objects. Rather the band serves as a frequency filter that iteratively removes the highest frequencies from a line. It controls for minimum amplitude of curves but not for a minimum spacing between objects including the line with itself. The band-width provides no direct evidence to indicate that legibility has been compromised, and because the band-width does not truly represent the width of the line there is no robust correspondence between a desired scale reduction and the band-width.

2.3 *Perkal's Epsilon Band*

An alternative to the rectangular band approximation of line width is the epsilon band. Perkal (1966b) described an epsilon band of a line L as the neighborhood N_ε' which includes all points on the plane not more than epsilon distance from L. Assuming a distance $d(x'p)$ between any point p and a point x on L the epsilon neighborhood can be described as:

$$N_\varepsilon(L) = \{p \mid d(x'p) \leq e\}$$

Figure 5 shows this neighborhood as a band of width 2ε enclosing line L plus two semicircles of radius ε. This epsilon band describes the locus of a line and an associated width. A new width is expressed simply by changing the value of ε. The value of ε is not a function of points along the line therefore the width is constant for the length of the line and the band does not reduce to zero unless ε is explicit-

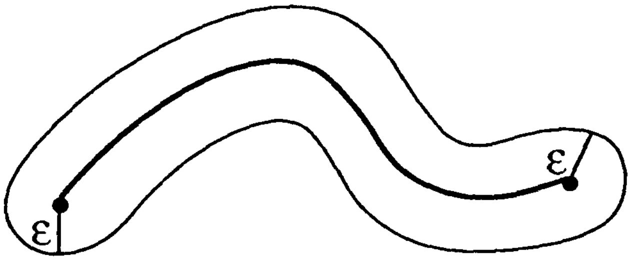


FIGURE 5. An example of a line with its epsilon band ε.

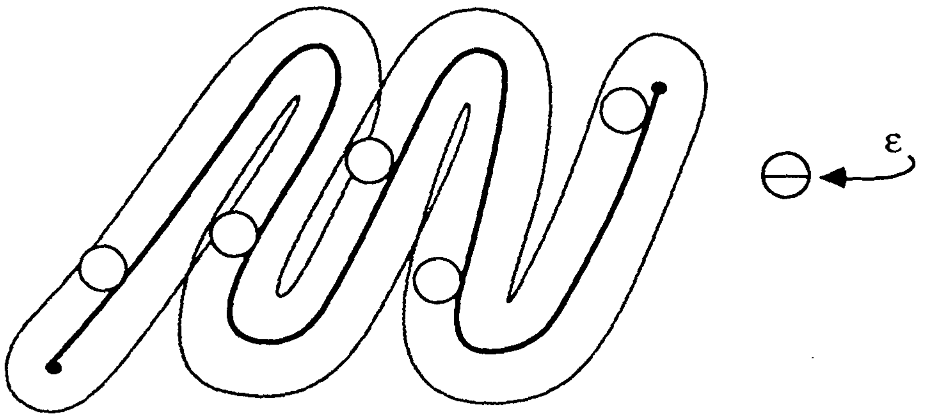


FIGURE 6. An example of a line which is not ε-convex.

ly set to zero. The epsilon band thus provides a model which more truly reflects the behavior of a cartographic line.

The most important phenomenon of the epsilon band for generalization, however, is the associated ε-convex set (Perkal 1966b). A line is ε-convex if every point on the line has a radius of curvature greater than or equal to ε. A line which is not ε-convex expresses the case in which a line having width 2ε folds back on itself as shown in Figure 6. Lack of ε-convexity in a line is therefore direct evidence that legibility has been compromised and generalization is required.

2.4 *Generalization Based on the Epsilon Band*

From the concept of ε-convexity, Perkal (1966a) demonstrated a generalization method. Perkal's method for generalizing a region M began with the placement of a circle of diameter ε inside the region. The circle was then rotated in such a manner that it remained completely inside the area. The ε generalization of M is thus the set of all points p having the property that they are contained within the circle of diameter ε, which can be completely included within the region M. This same process can be applied to the region outside M or the complement of M, designated M'. If the region is not ε-convex, there will be points in both M and M' that the circle will not touch. Examples of such areas are shown as shaded areas in Figure 7.

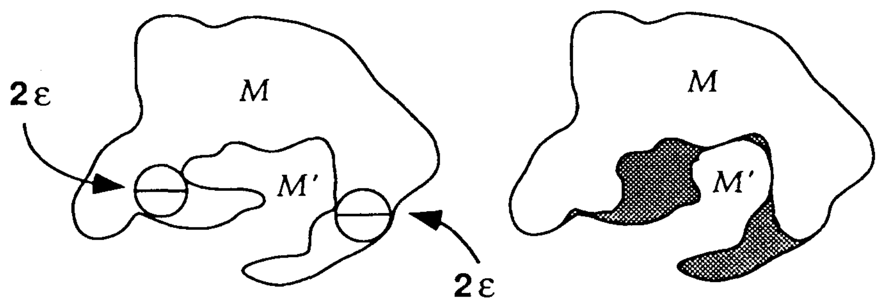


FIGURE 7. An epsilon generalization of Region M.

The result of the two generalizations is a new boundary for *M* and a new boundary for *M'*. Areas between the two boundaries are regions too small to accommodate a circle of diameter ε and thus lack ε-convexity. If this figure were to be drawn with a pen of width 2ε, the shaded areas will be areas of overlap or symbol collision and are the areas which will need to be generalized if they are to be legibly displayed. As applied to areas, lack of ε-convexity on the interior (or inability to place the circle in an area) signals that an area is too small to be legible.

As a generalization method, the procedure directly addresses the issue of legibility. By establishing ε-convexity where ε is equal to the minimum spacing threshold plus line width, it can assure legibility. The technique first controls for legibility by detecting areas of overlap and secondly by providing a corrective solution. If violations are discovered, ε-convexity can be achieved by employing either the interior or exterior trace of the circle.

With respect to defining importance, the algorithm is not definitive, but indirectly the geometric criteria of size and shape dictate importance. The general implication is that features with dimensions smaller than ε are not important and can therefore be removed. However, if certain features are deemed important, say by external user supplied criteria, the exterior and/or interior trace can provide a solution to achieve ε-convexity so features can be legibly retained.

2.5 *The* WHIRLPOOL *Implementation of Epsilon Generalization*

Perkal only applied his procedure manually. As Zoraster (1984) points out, the task of rolling a circle along a curve is not easily implemented in the computer. Brophy (1973) made one of the first attempts at a computer implementation of epsilon generalization. The ODYSSEY WHIRLPOOL algorithm (Dougenik 1980), while not a direct implementation of Perkal's concept, is a close approximation. The algorithm does not construct an ε envelope, but assures ε-convexity in the result. The algorithm operates on a distance threshold *t* comparable to ε. Clusters of points within *t* of each other are analyzed and points are removed from the clusters such that no two points are within *t* of each other and no points are moved more than *t* (Chrisman 1983). This assures that all resulting objects are at least *t* distance apart and that no line symbol of *t* width will intersect or overlap itself (i.e., ε-convexity is imposed). Figure 8 illustrates the operation of the algorithm.

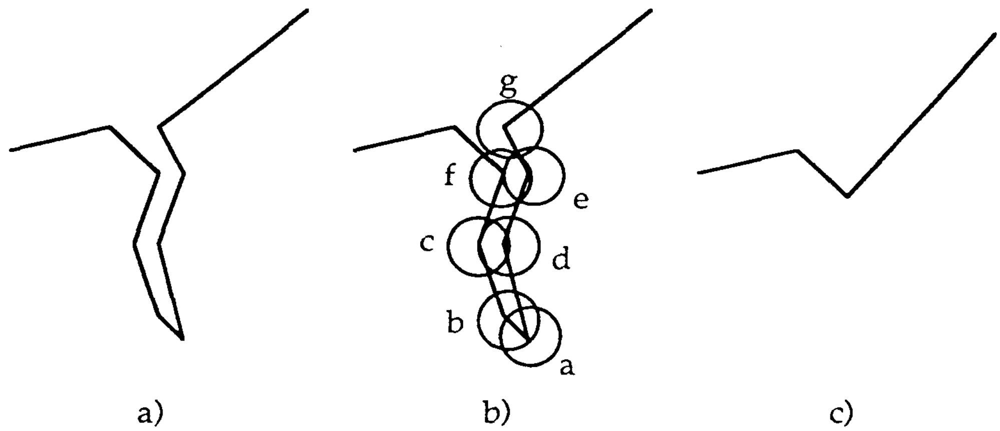


FIGURE 8. Generalization is accomplished by a cluster analysis. Clusters are formed of points within t of each. Poinst a and b form one cluster, c and d another, and points e, f and g, a third cluster. The cluster analysis requires that no points may be within t of one another and no points may move more than t. In the above clusters, only one point survives, and all other points in the cluster snap to that point. Any co-linear segments are then removed to create the final result.

Application of the WHIRLPOOL algorithm to generalization has been demonstrated by Chrisman (1983) and Beard (1988). The process results in the elimination of small areas and narrow features and the attachment of small features to larger ones where the intervening distance is sub-threshold. As it assures that no points are closer than a specified distance, it controls for sub-threshold areas, sub-threshold length segments, and sub-threshold spacings within and between objects. Because these thresholds are directly related to scale, the tolerance *t* can be set to these thresholds plus line width to produce a generalization appropriate for a desired scale reduction.

Although the WHIRLPOOL algorithm and the Douglas (1973) algorithm both use a tolerance distance as the criteria for retaining or removing information to generalize a representation, the end results are quire different. The WHIRLPOOL algorithm removes information based on distances within and between objects. The tolerance, as used by the algorithm, directly corresponds to the minimum size objects which can be displayed at a particular line width. The Douglas algorithm, as described above, filters information based on deviations from a trend line. The tolerance thus defines a frequency level and determines the frequencies which will be removed. It does not correspond to symbolized line width. The next section provides tests to illustrate the different performance of the two algorithms particularly as the degree of generalization is increased.

3. TESTS OF THE TWO METHODS

Three test data sets were used to demonstrate generalization performance and to test the behavior of the two methods with respect to different spatial patterns and levels of detail. The test sets included digital coastline data for North Carolina,

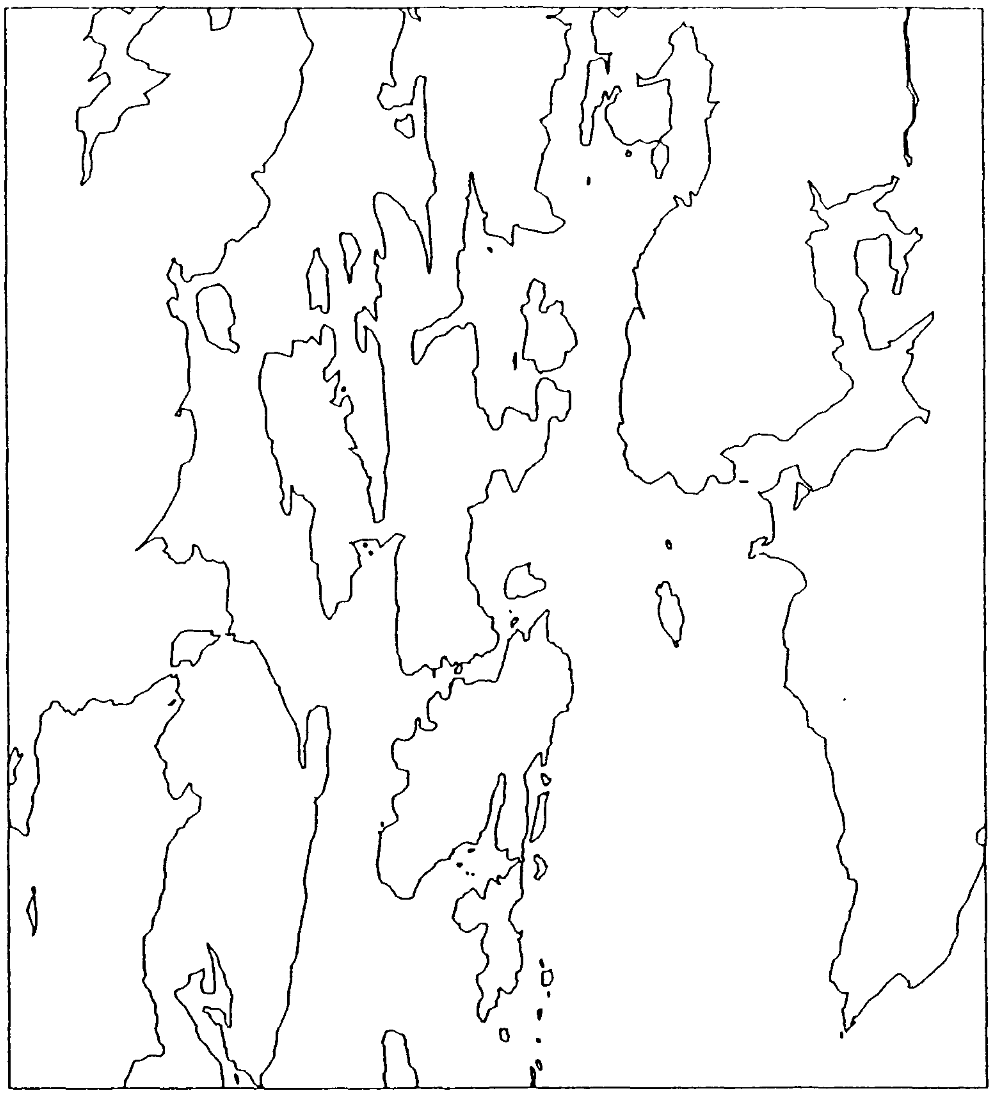


FIGURE 9. Detailed version of the Maine coastline around South Bristol.

South Carolina, and Maine provided by the National Ocean Service (NOS). These data were digitized by NOS from 1:10,000 to 1:80,000 scale charts. The different coastal geomorphology in each geographic area presents different challenges for the generalization methods. Figures 9–11 show the three test areas. Geomorphological differences in the three sections of coastline are reflected in differences in the distribution of graphic primitives used to represent them. The numbers of points in each test file were approximately the same, but configurations of the points are quite different. Table 1 shows the distribution of points, chains, maximum length

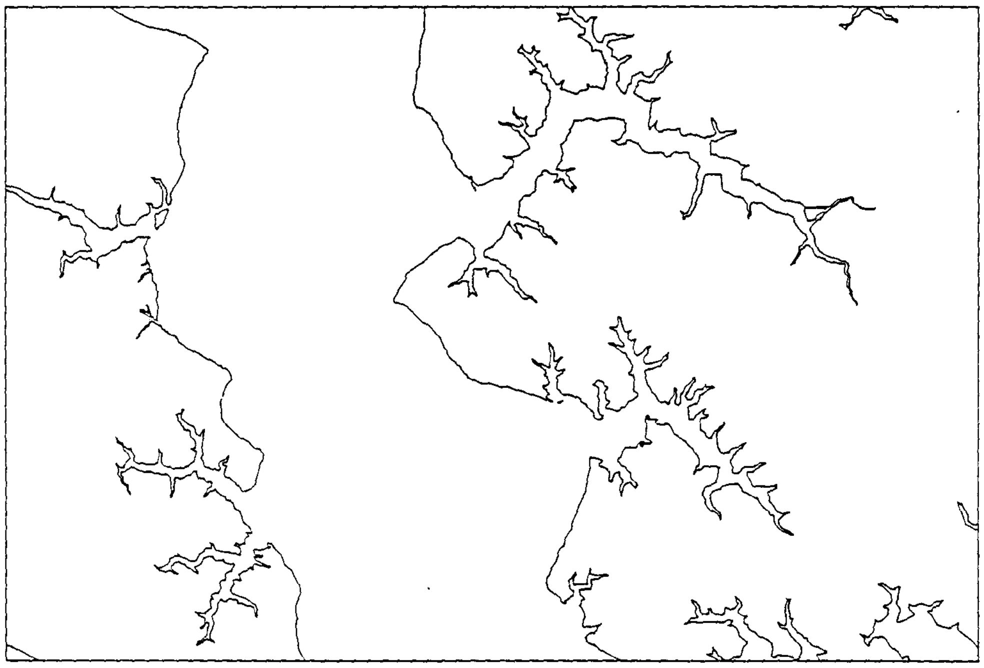


FIGURE 10. Detailed version of North Carolina coastline at the mouth of the Pango River on Pamlico Sound.

FIGURE 11. Detailed version of South Carolina coastline inside Isle of Palms on the inner coastal waterway.

TABLE 1. DISTRIBUTION OF POINTS, CHAINS AND POLYGONS FOR THE THREE TEST SECTIONS

Section	#Point	#Chains	#Polygons	Maximum Chain Length
Maine	2697	97	63	516
North Carolina	2598	26	12	823
South Carolina	2491	222	173	229

TABLE 2. SIZE DISTRIBUTION OF POLYGONS FOR THE THREE TEST SITES

	Polygon size (in hectares)					
Section	0–10	10–50	50–100	100–500	500–1000	>1000
Maine						
water	8	1	1	1	0	1
land	44	3	0	1	1	2
North Carolina						
water	1	1	0	2	0	1
land	4	0	0	0	1	2
South Carolina						
water	2	1	0	0	1	0
land	164	4	2	3	0	0

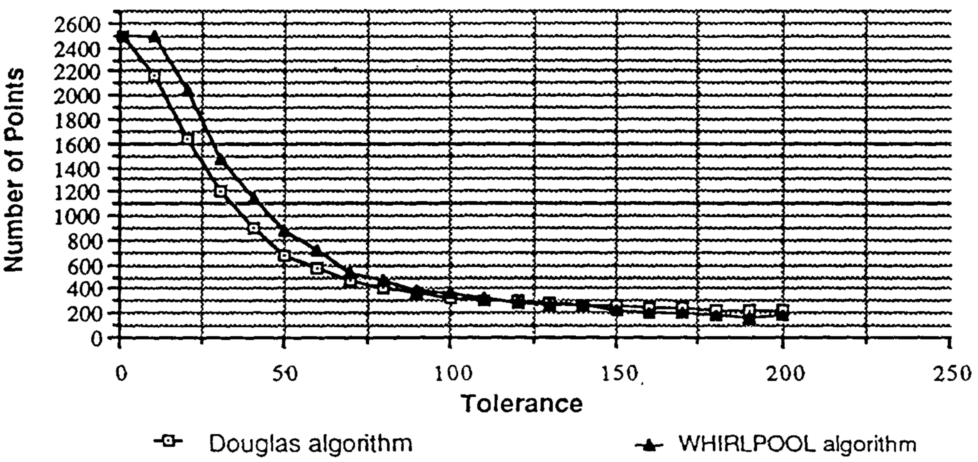
of chains, and numbers of polygons for the three test areas, and Table 2 illustrates the size distribution of land and water polygons for each test site.

The Maine coastline data contains polygons with an essentially bi-modal size distribution. This section also includes a number of long, complex chains. The challenge for a generalization process in this section is handling the small islands which lie close to each other and to the headlands. The North Carolina section contains the least number of polygons but the longest and most complex chains. This section of coastline tests the ability of the generalization methods to handle complex line detail without the interaction of other nearby objects. The South Carolina site contains the largest number of polygons and the least complex chains. This site includes many small islands which are very closely spaced. In fact, the spacing between islands is approximately the same as the interior dimensions of the islands. The interesting issue here is whether the islands should convert to a land mass, or the network of channels expand to become a solid body of water at a coarser resolution.

To perform the test, the data were converted from the NOS transfer format to topologically structured ODYSSEY files. The data, which were transferred in latitude/longitude, were transformed to Universal Transverse Mercator (UTM) coordinates using National Geodetic Survey (NGS) transformation routines. A local offset was subtracted from the UTM coordinates to maintain precision in subsequent processing. Each test site was processed in an identical manner.

The Douglas (D) and WHIRLPOOL (W) algorithms were applied independently to the three test data sets using progressively larger tolerances. The tolerances were increased in increments of ten meters for each iteration. Table 3 shows the results for the South Carolina data. For this data, small D tolerances removed points

TABLE 3. THE NUMBER OF POINTS removed from the South Carolina data as the D and W tolerances were increased from 10 to 100 meters.



rapidly up to forty meters, at which point the effect of the tolerance leveled off. The W algorithm actually generated points for the first ten meter tolerance. It then removed points at a slower rate over the same tolerance range up to 120 meters. Beyond 120 meters, the W tolerance removed points at a slightly faster rate than the D algorithm. For the South Carolina data set, D tolerances larger than forty meters began to generate line crossings. The graphic results of these tests for the 50, 100, 150, and 200 meter tolerances are displayed in Figures 12 through 15. The tolerances are shown as small circles in the lower left corners. It is important to notice that many small channels and inlets are retained by the Douglas algorithm. In the WHIRLPOOL results we can see that small channels and islands which would not accommodate a circle of diameter *t* have been removed. The other effect to notice is that the algorithm breaks channels and peninsulas at locations too narrow to accommodate a circle of *t* diameter. This is, in most cases, an undesirable effect of the algorithm.

Table 4 summarizes the results of the Douglas and WHIRLPOOL algorithms applied to the North Carolina data. Similarly, small D tolerances removed points quite rapidly up to about fifty meters. At this tolerance, the rate of point elimination dropped off and beyond 150 meters the number of points removed by each larger tolerance began to converge with the numbers eliminated by the W algorithm. The graphic results of both algorithms for 50, 100, 150, and 200 meter tolerances are shown in Figures 16–19. The tolerances are shown as small circles in the lower center of the plots. We can see in these results that the WHIRLPOOL algorithm iteratively removes the smallest/most narrow channels while the Douglas algorithm retains these and produces very spiky results.

Table 5 summarizes the results of applying the two algorithms to the Maine data. The results are quite similar to those for the North Carolina data. The Douglas algorithm removes points rapidly for small tolerances but the removal rate

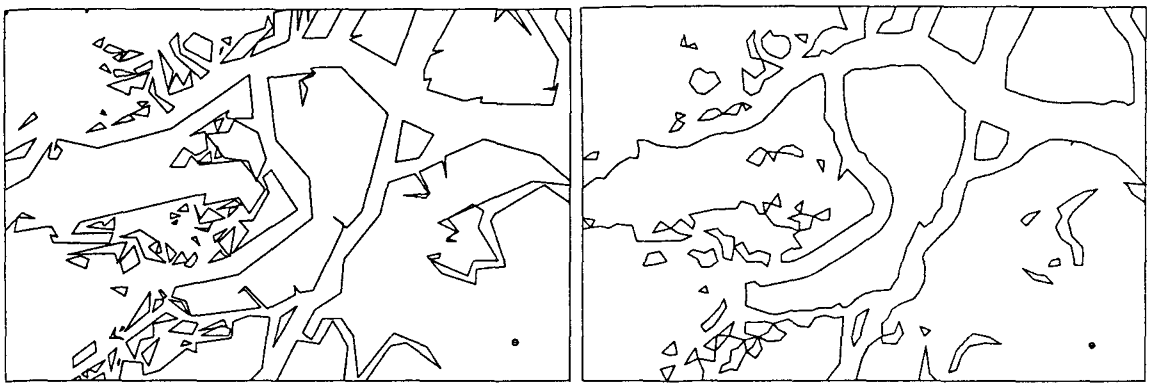


FIGURE 12. South Carolina coastline plotted at 1:50,000 scale: **a**. Douglas algorithm – 50 meter tolerance; **b**. WHIRLPOOL algorithm – 50 meter tolerance.

Note: Figures 12 to 19 are published at a reduction of 50 %.

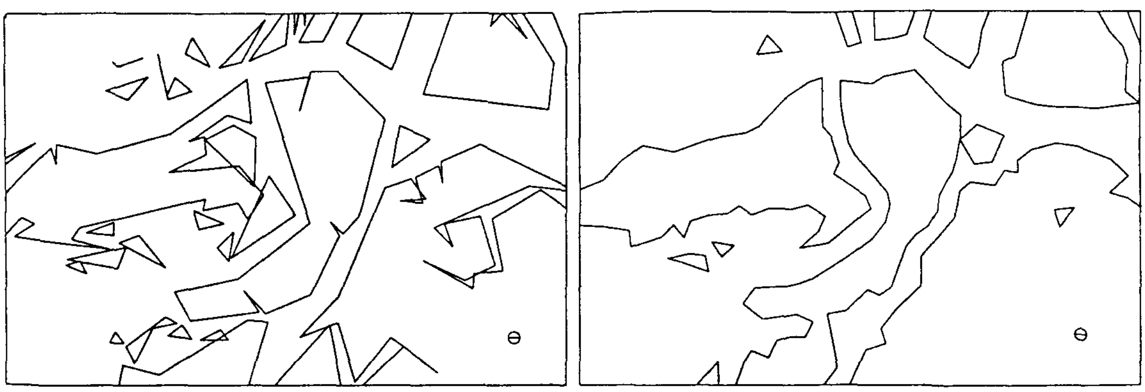


FIGURE 13. South Carolina coastline plotted at 1:50,000 scale: **a**. Douglas algorithm – 100 meter tolerance; **b**. WHIRLPOOL algorithm – 100 meter tolerance.

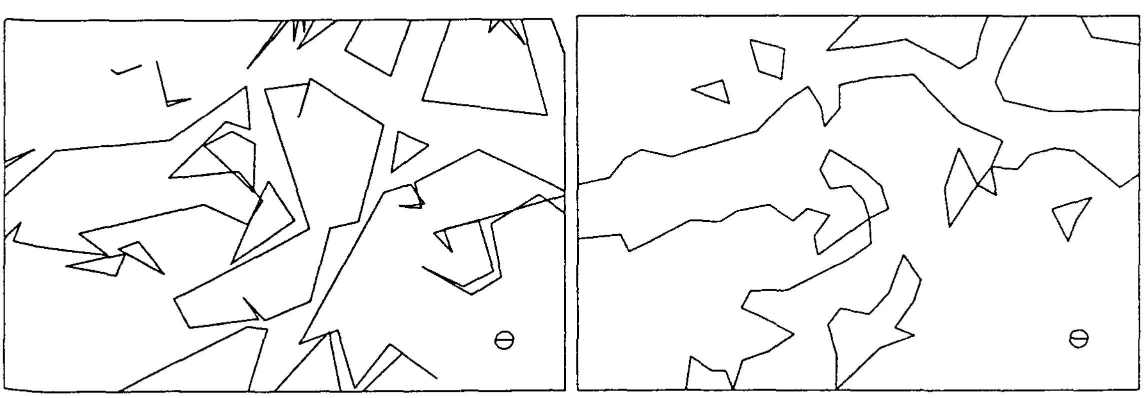


FIGURE 14. South Carolina coastline plotted at 1:50,000 scale: **a**. Douglas algorithm – 150 meter tolerance; **b**. WHIRLPOOL algorithm – 150 meter tolerance.

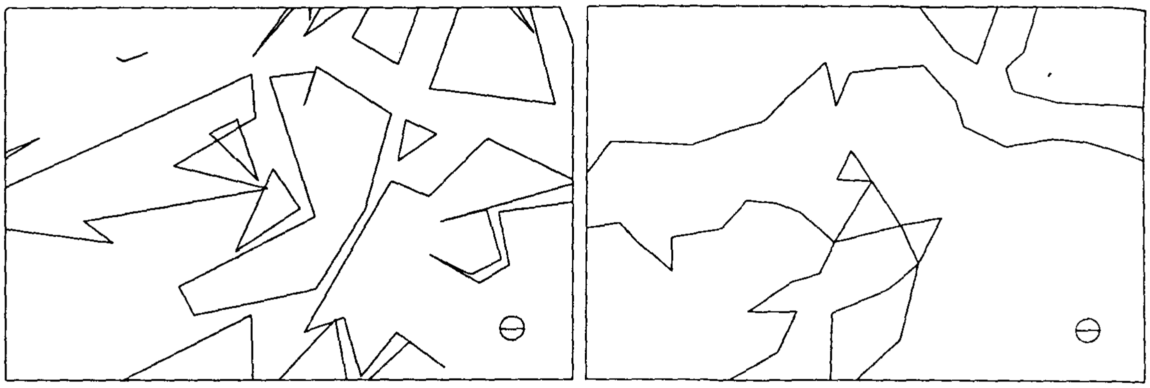


FIGURE 15. South Carolina coastline plotted at 1:50,000 scale: **a.** Douglas algorithm – 200 meter tolerance; **b.** WHIRLPOOL algorithm – 200 meter tolerance.

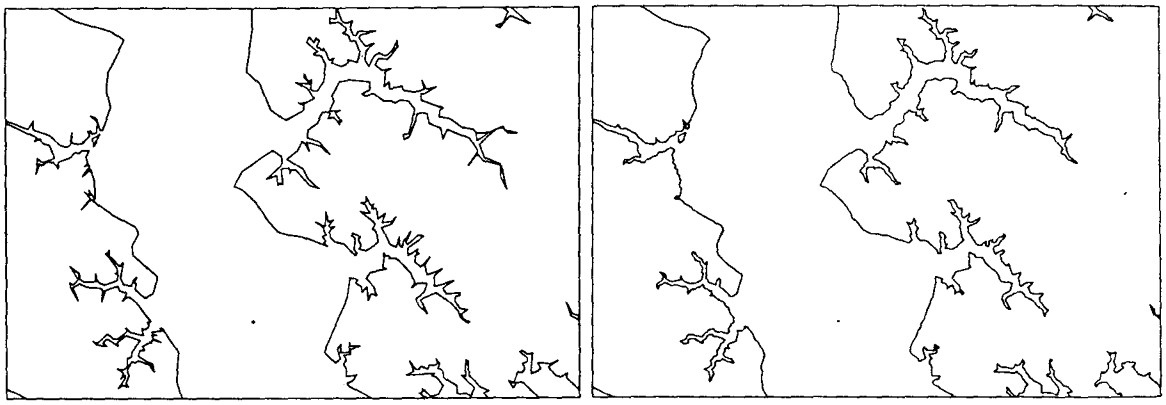


FIGURE 16. North Carolina coastline plotted at 1:50,000 scale: **a.** Douglas algorithm – 50 meter tolerance; **b.** WHIRLPOOL algorithm – 50 meter tolerance.

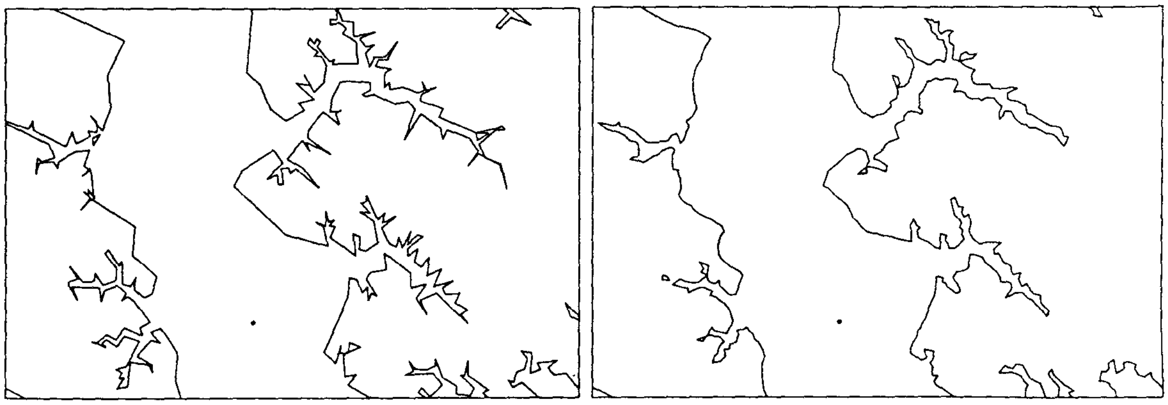


FIGURE 17. North Carolina coastline plotted at 150,000 scale: **a.** Douglas algorithm – 100 meter tolerance; **b.** WHIRLPOOL algorithm – 100 meter tolerance.

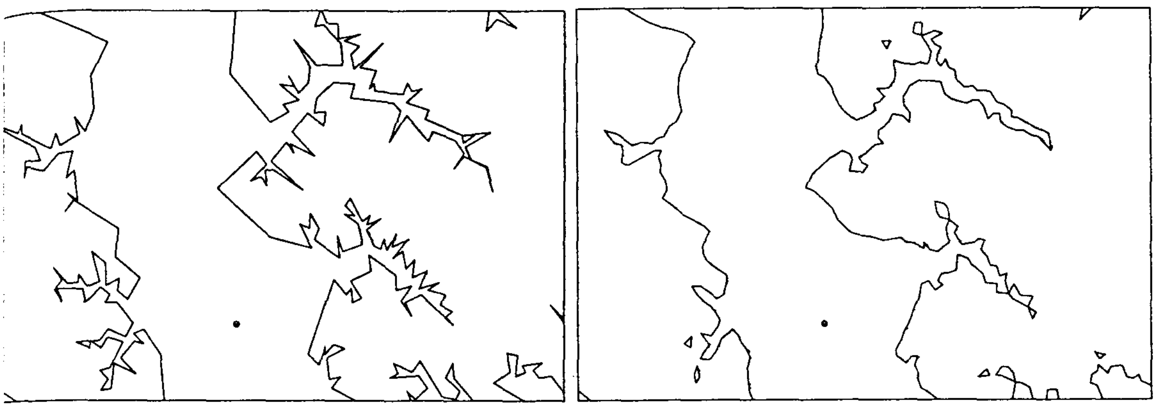


FIGURE 18. North Carolina coastline plotted at 150,000 scale: a. Douglas algorithm – 150 meter tolerance; b. WHIRLPOOL algorithm – 150 meter tolerance.

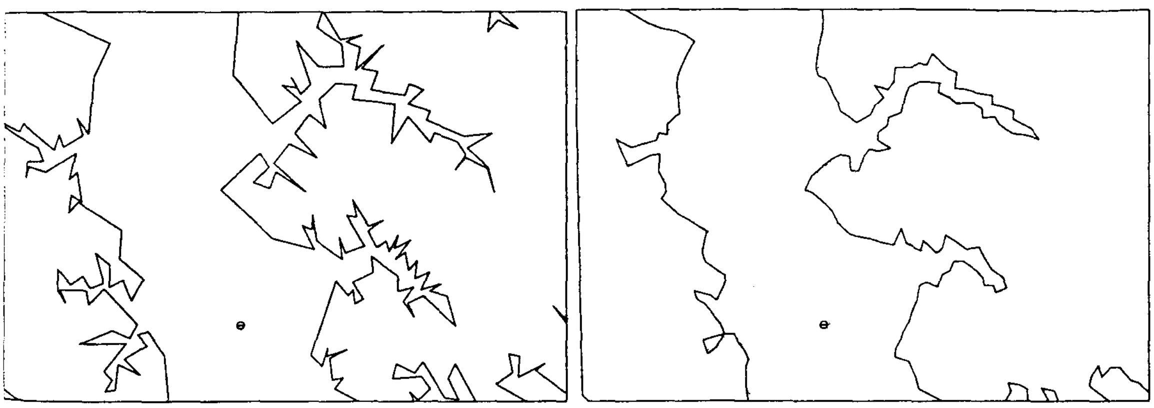


FIGURE 19. North Carolina coastline plotted at 150,000 scale: a. Douglas algorithm – 200 meter tolerance; b. WHIRLPOOL algorithm – 200 meter tolerance.

levels off at tolerances over 100 meters. The graphic results of both algorithms for 50, 100, 150, and 200 meter tolerances are shown in Figures 20–23. The tolerances are shown as small circles in the lower left corners of the plots. In this case, both algorithms iteratively removed the smallest islands. The Douglas algorithm, however, retained many of the narrow channels and spits (see in particular the narrow channel in the upper right hand corner), while the WHIRLPOOL algorithm consistently removed those with dimensions less than *t*.

3.1 *Comparison of results against manual generalizations*

To evaluate the performance of the two generalization methods for large reductions in scale, manually generalized 1:250,000 coastline data from NOS were used for comparison. The 1:250,000 scale data, provided in digital form by NOS, were also converted to ODYSSEY format and transformed from latitude/longitude to UTM coordinates. The transformations for each test area used the same local offsets

TABLE 4. THE NUMBER OF POINTS removed from the North Carolina data as the D and W tolerances were increased from 10 to 200 meters.

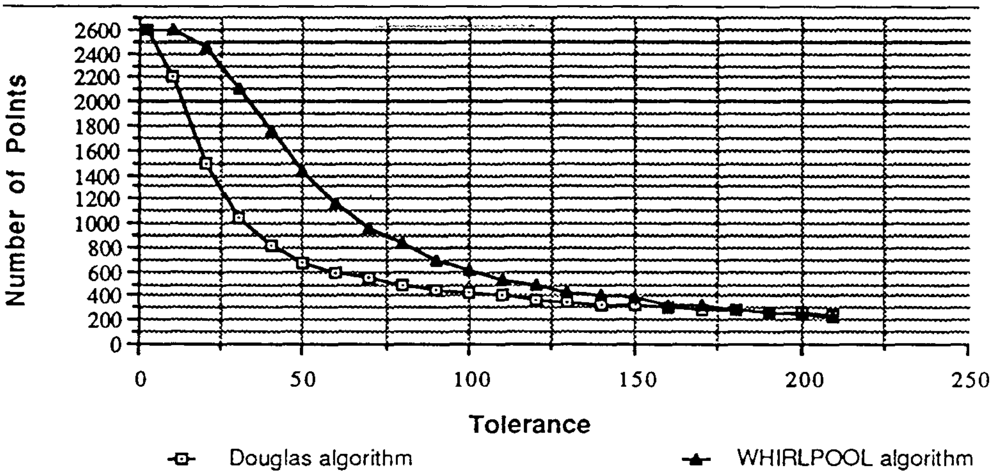
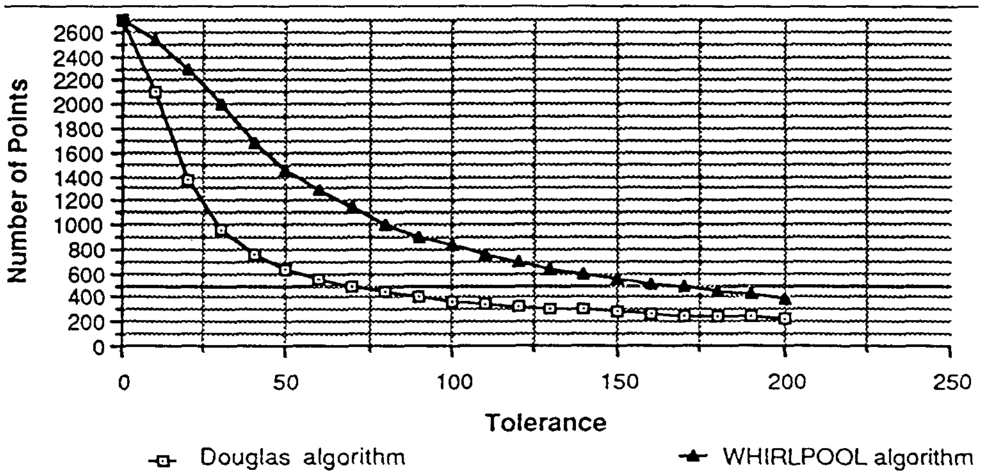


TABLE 5. THE NUMBER OF POINTS removed from the Maine data as the D-P and W tolerances were increased from 10 to 200 meters.



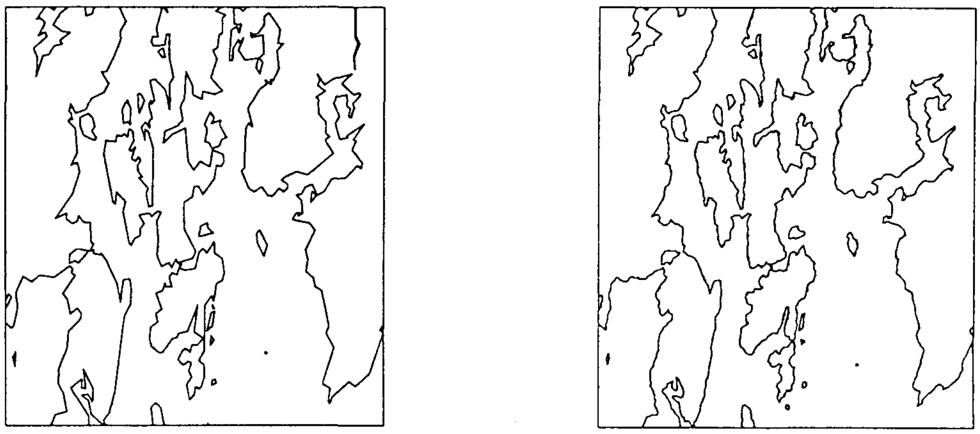


FIGURE 20. Maine coastline plotted at 1:125,000 scale: **a.** Douglas algorithm – 50 meter tolerance; **b.** WHIRLPOOL algorithm – 50 meter tolerance.

Note: Figures 20 to 23 are published at a reduction of 50%.

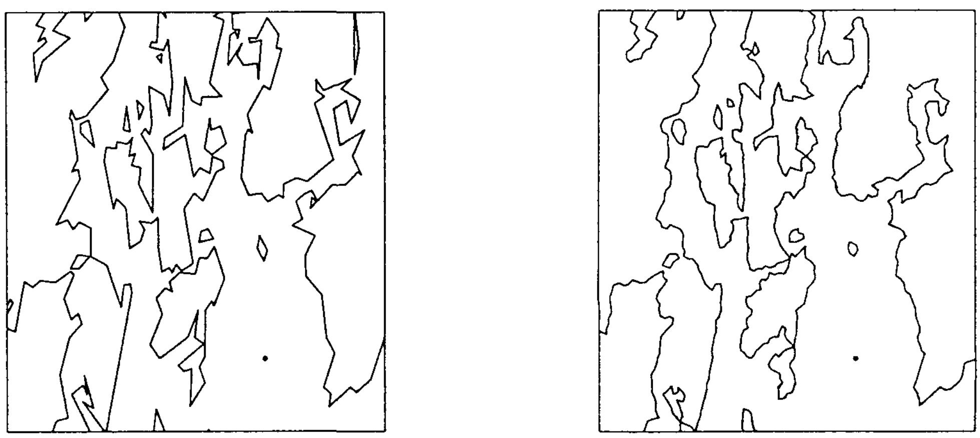


FIGURE 21. Maine coastline plotted at 125,000 scale: **a.** Douglas algorithm – 100 meter tolerance; **b.** WHIRLPOOL algorithm – 100 meter tolerance.

applied in the transformation of the corresponding detailed data files. Figures 24–26 show the manually generalized versions for each test area. These data sets provided evidence of the level of detail and types of features a cartographer deemed important for a 1:250,000 scale representation.

Performance of the comparison first required the generation of comparable results from the detailed data. The most detailed portions (1:10,000 scale) of the test sites exceeded the manually generalized test data sets by a factor of 25. The average reduction over the various scales, was approximately a factor of 13; in either case a sizeable scale reduction. This raised the question of what tolerance should be applied to generate a 1:250,000 scale result. Using legibility criteria which has a direct relationship to scale, a tolerance can be computed as the minimum separation requirement between objects plus symbolized line width. If we assume that lines will be drawn with a .2 mm pen and the minimum spacing between lines

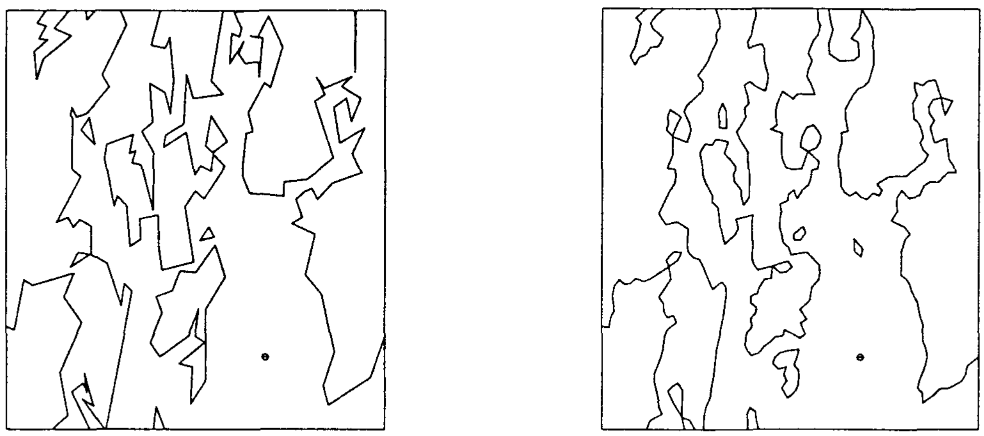


FIGURE 22. Maine coastline plotted at 125,000 scale: **a.** Douglas algorithm – 150 meter tolerance; **b.** WHIRLPOOL algorithm – 150 meter tolerance.

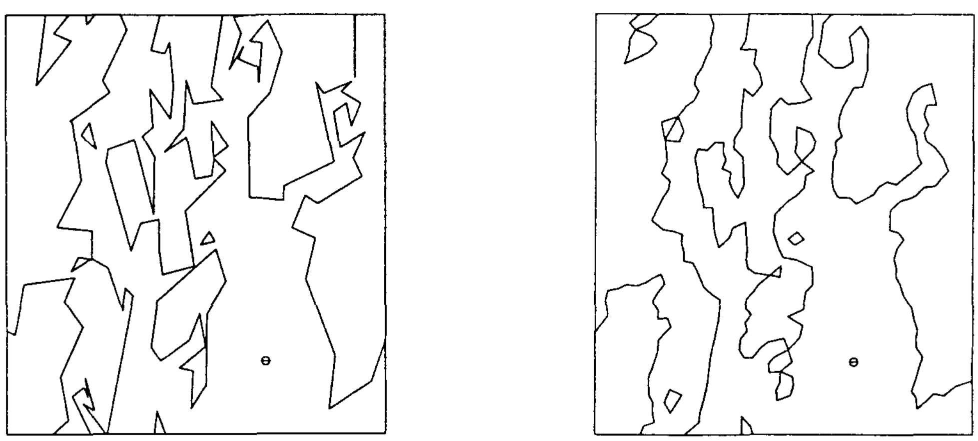


FIGURE 23. Maine coastline plotted at 125,000 scale: **a.** Douglas algorithm – 200 meter tolerance; **b.** WHIRLPOOL algorithm – 200 meter tolerance.

must be .25 mm (based on the minimum dimension requirements from Robinson et al 1984) then we can compute *t* to be .45 mm or 112.5 meters on the ground at 1:250,000 scale. Because there is no correspondence between a target scale and the Douglas algorithm tolerance, an appropriate tolerance could not be estimated for this algorithm. For comparative purposes, however, the 112.5 tolerance was rounded to 112 meters and applied to each data set. Figures 27–29 show the results for each test area. These were compared visually against the manually generalized 1:250,000 scale data, an approach supported by Visvalingam and Whyatt (1990). Comparison of the 112 meter WHIRLPOOL generalizations against the 1:250,000 scale versions first demonstrates that this tolerance did generate comparable results. The 112 meter WHIRLPOOL generalization of the North Carolina data, in particular, is remarkably similar to the manually generalized version. Almost identical features

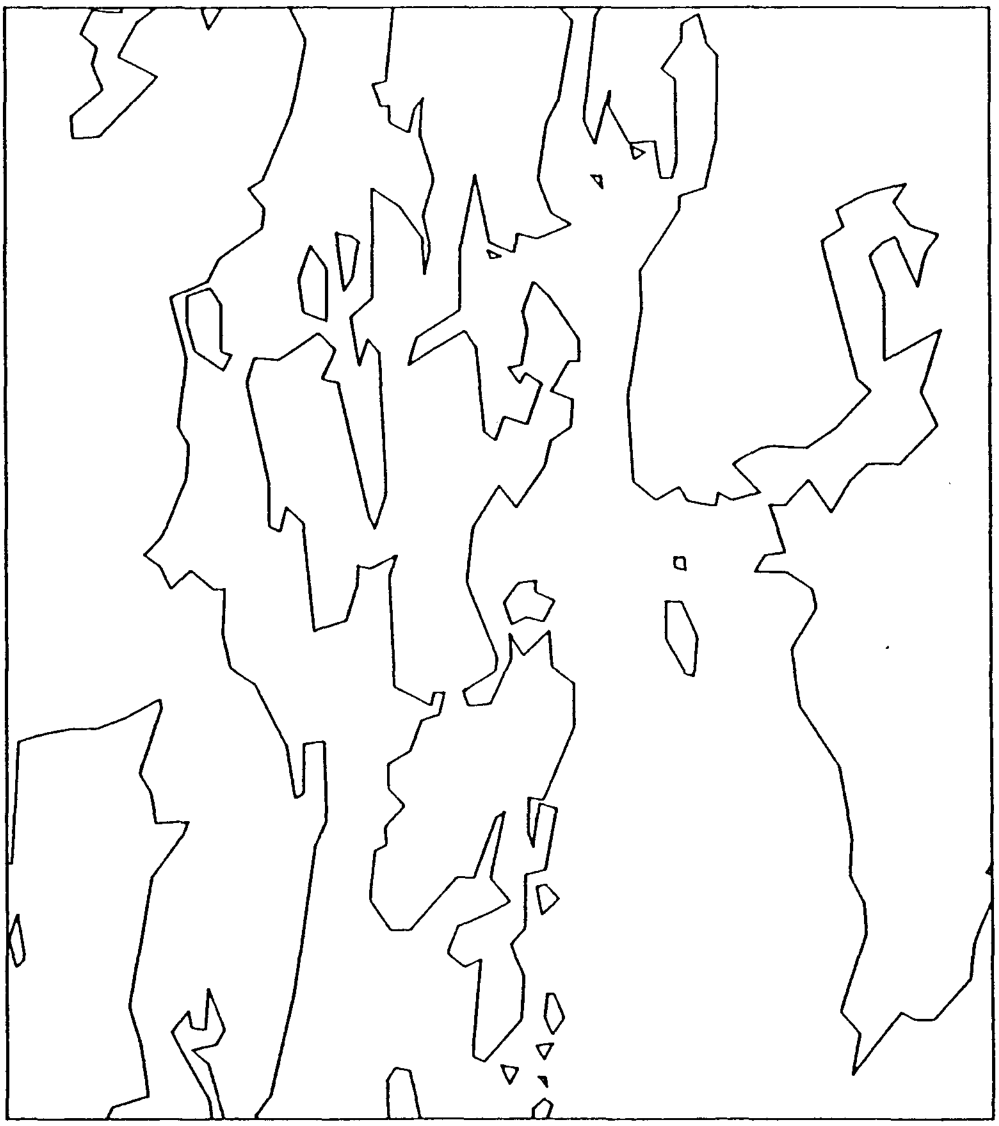


FIGURE 24. Maine coastline manually generalized by NOS for 1:250,000 scale.

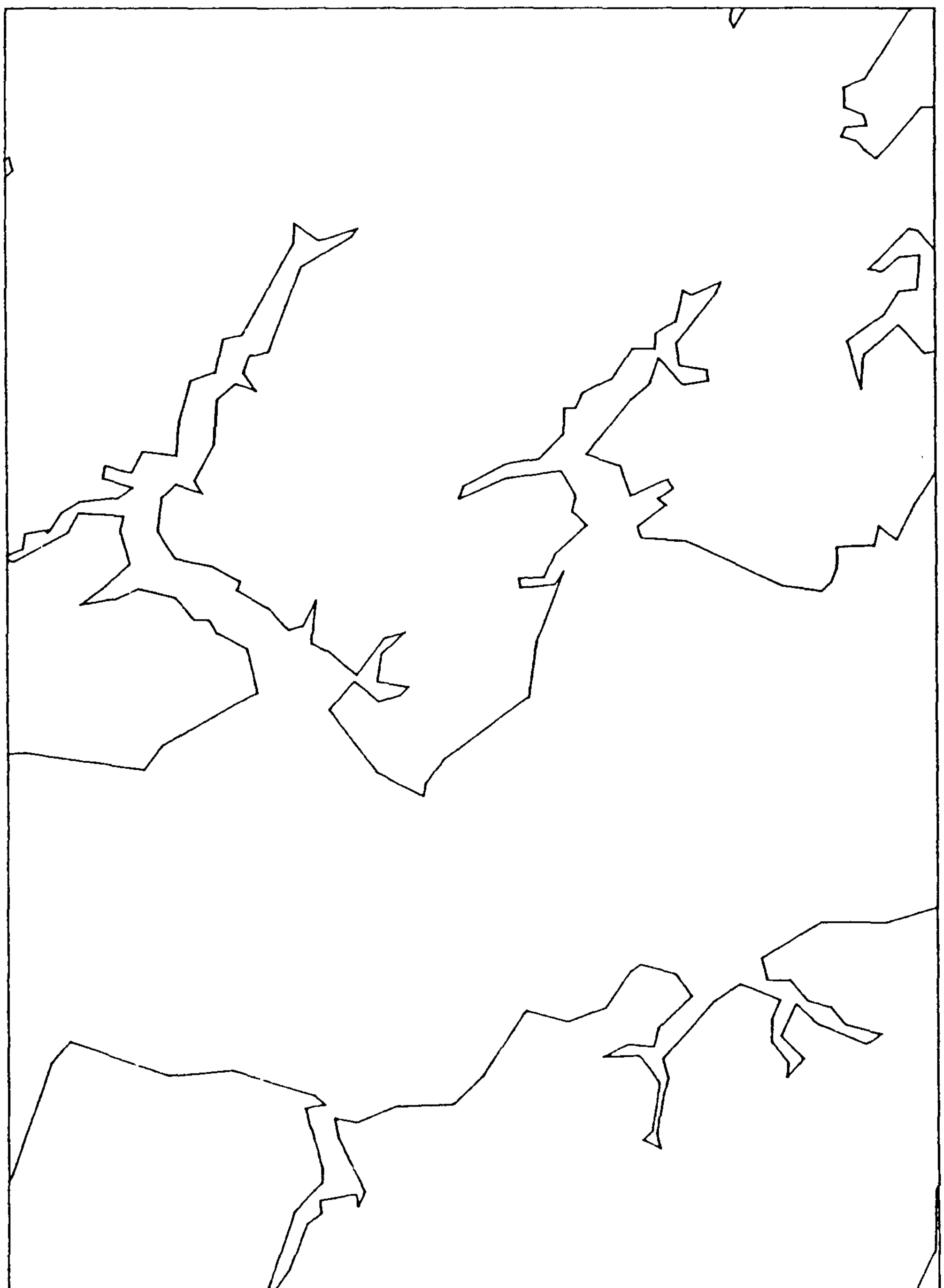


FIGURE 25. North Carolina coastline manually generalized by NOS for 1:250,000.

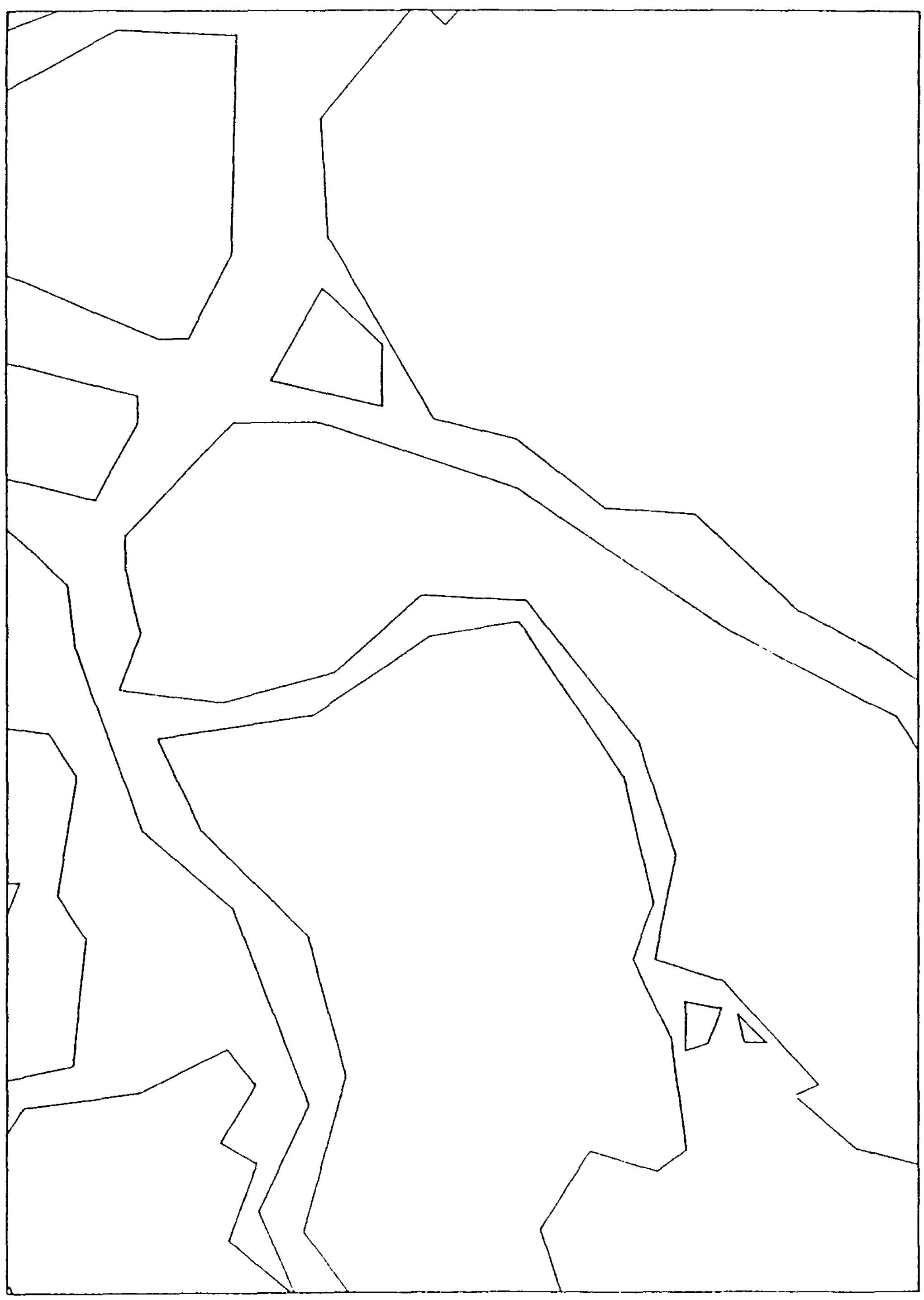
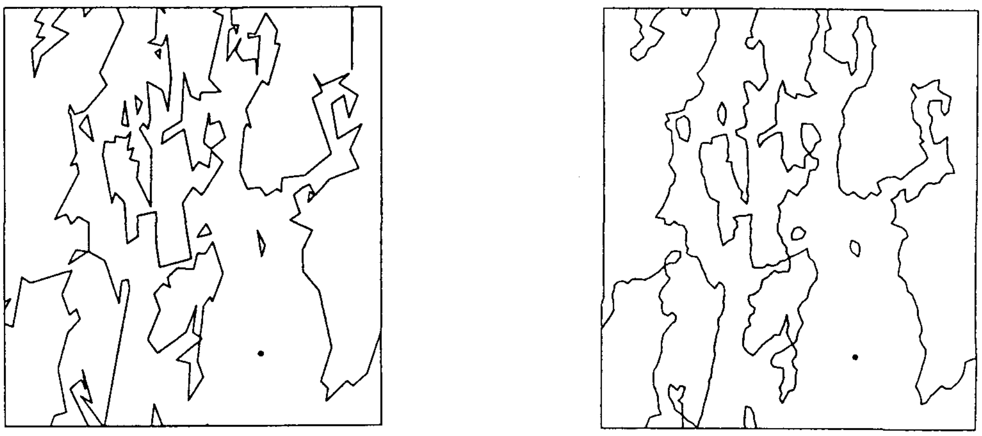


FIGURE 26. South Carolina coastline manually generalized by NOS for 1:250,000 scale.



Note: Figures 27 to 29 are published at a reduction of 50%.

FIGURE 27. Maine coastline plotted at 1:125,000 scale: **a.** Douglas algorithm – 75 meter tolerance; **b.** WHIRLPOOL algorithm – 112.5 meter tolerance.

FIGURE 28. North Carolina coastline plotted at 1:150,000 scale: **a.** Douglas algorithm – 160 meter tolerance; **b.** WHIRLPOOL algorithm – 112.5 meter tolerance.

were removed by the WHIRLPOOL algorithm as were removed by the cartographer. The Douglas algorithm, in contrast, retained almost all of the small estuaries as sharp points and these features persisted even under the larger tolerances of 150 and 200 as shown in Figures 18 and 19. At the 200 meter tolerance the Douglas algorithm result has approximately the same number of points as the manually generated 1:250,000 scale version, so even using this criteria it is obvious that the algorithm does not produce results comparable to the cartographer's.

In the South Carolina test, the WHIRLPOOL algorithm was on the right track. It merged many of the smaller islands into larger blocks of land which was the approach taken by the cartographer. The Douglas algorithm also removed many of the smaller islands but retained many of the small estuaries. The WHIRLPOOL algorithm merged many of the small islands with larger islands so it mimicked the

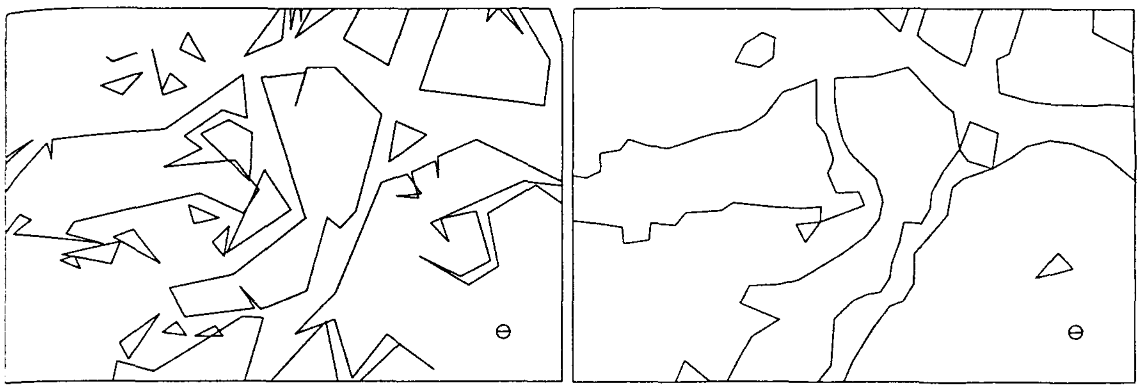


FIGURE 29. South Carolina coastline plotted at 1:50,000 scale: **a.** Douglas algorithm – 0 meter tolerance; **b.** WHIRLPOOL algorithm – 112.5 meter tolerance.

topological changes made by the cartographer. The Douglas algorithm does not exhibit this behavior and has the problem that a number of small islands overlap each other.

For the Maine data, neither of the results succeeded entirely in retaining the same information as retained by the cartographer. The same islands are retained by both algorithms, but the cartographer enlarged and retained additional islands. These were probably retained because they are important to navigation (solid rock outcrops in what are generally navigable waters). The cartographer also widened a number of peninsulas. The Douglas algorithm tended to reduce these to spiky points and WHIRLPOOL algorithm eliminated a number of them. Also,the cartographer merged a number of islands with nearby peninsulas. The WHIRLPOOL algorithm did merge islands with nearby land, but not always to the same mainland points as used by the cartographer. The Douglas algorithm does not do this.

In all three test cases, the WHIRLPOOL algorithm can be seen to systematically remove features which are too small to accommodate a 112 meter circle. In this respect it satisfied the legibility requirement, but it cannot in all cases replicate the manual results. The Douglas algorithm quite consistently retained features despite the fact that many would be illegible if actually displayed at 1:250,000 scale. In the North and South Carolina results, even the smallest channels were retained, which is a result of the algorithm determining these features to be important within the context of the line. The manual results document the removal of these features and provide evidence that these features were not deemed important by the cartographer for the 1:250,000 scale.

4. CONCLUSION

In terms of satisfying the original objectives of retaining essential or important characteristics, the Douglas algorithm has been shown to be reasonably successful for small tolerances or small scale reductions. For large scale reductions it is not effective. The definition of importance is limited to the context of the line which is satisfactory for small scale reductions, but inappropriate for larger scale reductions which need to consider larger spatial neighborhoods. The significant implication is that this definition of importance is scale bound. The algorithm also does not address the issue of legibility. The band, which is used as a basis for generalization, does not simulate cartographic line width and does not assist in locating and resolving line collisions. A third limitation lies in lack of a direct connection between the algorithm tolerance and a target scale.

The epsilon band, in contrast, simulates the behavior of a cartographic line, and the related concept of ε-convexity provides a basis for discovering legibility violations. It thus has the ability to target specific locations where generalization is required. The WHIRLPOOL algorithm, as an implementation of this concept, has the ability to generate discrete approximations of ε-convex results and thus preserves legibility. The other important factor is that the associated tolerance can be directly related and computed from a desired symbol width and the minimum separation between objects dictated by a target scale. The success of the algorithm in retaining important or essential characteristics is also supported in general by comparison with cartographers' manual generalizations. The algorithm removes small features which are frequently not considered important at small scales. In fact, the situations in which it deviated from the cartographer's results were typically cases in which features were sub-tolerant but the cartographer had exaggerated them to retain legibility rather than eliminating them.

The WHIRLPOOL algorithm, however, is not an ideal generalization processor. Because it currently uses only simple geometric criteria, its results are not always predictable. The important point is that the algorithm is able to identify areas which require generalization. With additional intelligence built into the process, it could become more sophisticated in resolving the conflicts instead of simply eliminating the offending features.

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RÉSUMÉ La théorie de la ligne cartographique (Peucker 1975) décrit la largeur comme étant la caractéristique essentielle d'une ligne cartographique. Les représentations informatiques ont eu tendance à ignorer cet attribut de base; dans le contexte de la généralisation, l'oubli est nuisible. La théorie soutient qu'un ensemble de bandes délimitatrices représente le caractère cartographique de la largeur et supporte la généralisation. L'algorithme de Douglas, toujours l'un des algorithmes les plus communément employés pour généraliser les représentations numériques, utilise ce modèle. Les travaux du mathématicien polonais Perkal procurent les bases d'un autre modèle de largeur de ligne cartographique ainsi que d'une technique de généralisation différente. Cet article examine l'efficacité des deux modèles pour capturer la largeur d'une ligne cartographique et comment ils réussissent à produire des résultats généralisés, particulièrement lors de la réduction d'échelles plus grandes. Les deux techniques sont évaluées sur leur aptitude à satisfaire deux objectifs: capter les caractéristiques essentielles et reconnaissables d'éléments géographiques et créer des représenttions qui peuvent être affichées lisiblement à des échelles plus petites. L'arcticle compare le comportement des deux méthodes lors de leur application à des données numériques de rivage.

ZUSAMMENFASSUNG 'Die Theorie einer Kartographischen Linie' (Peucker 1975) beziechnet die Breite als wesentliches Merkmal einer kartographischen Linie. Digitale Darstellungen tendierten dahin, dieses grundsätzliche Kennzeichen zu mißachten, und im Zusammenhang der Generalisierung ist dieses Versehen abträglich. Die Theorie behauptet, daß ein Satz umschließender Bande den kartographischen Charakter der Breite erfaßt und die Generalisierung unterstützt. Der Douglas-Algorithmus, der immer noch einer der gebräuchlichsten Algorithmen zur Generalisierung digitaler Darstellungen ist, verwendet dieses Modell. Das Werk des polnischen Mathematikers Perkal liefert den Grundstock für ein anderes Modell kartographischer Linienbreite und eine andere Generalisierungsmethode. Der vorliegende Aufsatz untersucht, wie effektvoll beide Modelle die kartographische Linienbreite einfangen und generalisierte Ergebnisse hervorbringen, besonders für größere Maßstabsreduzierungen. Beide Verfahren werden hinsichtlich ihrer Fähigkeit eingestuft, zwei Zielvorgaben zu erfüllen: die wesentlichen und erkennbaren Eigenschaften geographischer Kartenmerkmale zu erfassen und Darstellungen anzubieten, die in kleineren Maßstäben leserlich gezeigt werden können. Der Aufsatz vergleicht das Verhalten der beiden Verfahren im zuge ihrer Anwendung bei digitalen Küstenliniendaten.