The Scope and Conceptual Content of Analytical Cartography

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ABSTRACT: Over the last three decades analytical cartography has grown from Tobler's concept of "solving cartographic problems" into a broader and deeper scientific specialization that includes the development and expansion of analytical/mathematical spatial theory and model building. In many instances Tobler himself has led the way to these new insights and developments. Fundamental concepts begin with Tobler's cartographic transformations; Nyerges' deep and surface structure and data levels; and Moellering's real and virtual maps; the sampling theorem; and concepts of spatial primitives and objects. This list can be expanded to include additional analytical concepts such as spatial frequencies, spatial surface neighborhood operators, information theory, fractals, Fourier theory, topological network theory, and analytical visualization, to name a few. This base of analytical theory can be employed to analyze and/or develop such things as spatial surfaces, terrain analysis, spatial data schemas, spatial data structures, spatial query languages, spatial overlay and partitioning, shape analysis, surface generalization, cartographic generalization, and analytical visualization. More analytical uses of theory, strategies of analysis, and implementations are being developed and continue to multiply as the field continues to grow and mature. A primary goal is to expand the mathematical/analytical theory of spatial data analysis, and theory building and analytical visualization as analytical cartography takes its place in the geographic information sciences. The research future for this area appears very bright indeed.

KEYWORDS: Analytical cartography, map transformations, spatial theory, real and virtual maps, deep and surface structure, sampling theorem, spatial frequencies, analytical operators, information theory, fractals, Fourier theory, spatial overlay, Warntz network theory, map generalization, analytical visualization, geographic information science

Introduction

Analytical cartography was defined by Professor Waldo Tobler in the 1960s as a more refined approach to cartography articulated via the notion of "solving cartographic problems" (Tobler 1966). As such, it offers a more analytical, conceptual, and mathematical approach to the field of cartography, and even to some related areas. This work attempts to assess the breadth and depth of the field by examining a number of research topics in analytical cartography and associated fields. However, in a survey of a topic of this magnitude, one cannot include all research topics in the field, and one cannot include all relevant references. One must by necessity choose and select research areas to review, and then provide only a selection of the wealth of references that are possible. It is hoped that this work constitutes a reasonable survey of the breadth and depth of the field of analytical cartography.

The formal conceptualization of analytical cartography began with Tobler's dissertation in 1961 where he initiated his concepts of map transformations of geographic space (Tobler 1961). Tobler later extended his 1961 conceptualization of cartographic transformations to explain the traditional cartographic map creation and reading process, later articulated by Muehrcke (1973). This transformational view of cartography was further refined by Tobler in 1979 by stating that cartographic transformations "...occur in map interpolation, filtering, and generalization..." (Tobler 1979a). More recent advances in the transformational approach to analytical cartography revolve around the work of Nyerges (1980), Clarke (1990) and Chrisman (1999).

At roughly the same time, the field of traditional cartography was thrust into a conceptual turmoil with the development of new cartographic products, many of which were digital, such as CRT display images, digital terrain models, and spatial databases. These products did not fit into the then conventional definition of traditional hard copy cartography. This dilemma was articulated clearly by Morrison (1974) in the first article in the premier issue of *The American Cartographer*, where he called for an expanded and extended definition of what constitutes a map. Moellering, who was confronted

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with a similar need, initiated research on the problem highlighted by Morrison.

Goals of Analytical Cartography

During this time, Prof. Tobler (1966), while discussing the concepts of analytical cartography, would talk about the half-life of theory versus the half-life of technique as the reason why one wanted a quantitatively and mathematically based field of study. He estimated that the half-life of technique, which is what most people in cartography did in those days, was about three to five years, whereas the half-life of theory was about 15 to 20 years. This means that if one focuses one's efforts on the conceptual and theoretical aspects of cartography, the likelihood of developing better insights into the field is increased. This also means that research work in the field could be replicated elsewhere, and benefit from many of the other aspects of a normative science. So it is that analytical cartography has mainly focused on the quantitative and mathematical relationships inherent in the broadly defined field of cartography. This approach to cartography is similar to the mathematical mode of inquiry (MMOI) advocated by Casetti (1999) for human geography.

Two additional goals of the field are to extend the body of existing spatial theory, such as develop further uses and extensions of the sampling theorem, and other theory like it, and to develop new conceptual theory, such as Moellering's (1977b) concept of real and virtual maps and their applications. Several efforts in the field (e.g., Müller's (1991) book on the cartographic research agenda) have begun to explicitly recognize these needs, some of which have touched on analytical cartography. Moellering (1991c; 1994) also recognized that the effort to develop spatial data standards identified a distinct need to expand the analytical theory so as to fill the huge conceptual gaps in it. An effort was begun by Moellering (1991a, 1991b) to fill in some of these gaps by organizing and editing a special content issue of *Cartography and Geographic Information Systems* dedicated to Analytical Cartography. Cartography needs to make a major push to expand its theoretical base because it is clear now that no matter how powerful, the computer may not be able to empirically or heuristically solve certain classes of numerical problems. This problem is discussed by Saalfeld in this issue, in his article on complexity and intractability of algorithms and computer processing. He suggests that problems that are NP complete or NP-hard may not solved by merely adding more computer horsepower (Saalfeld 2000). Williams' (1996) view of a "new era in cartography" in Australia emphasizes expanding and adapting institutional changes in a similar context. It is clear that such changes are necessary to accompany the expanded need for further developing the cartographic and spatial theory discussed herein.

Analytical Cartography in Relation to Other Branches of Cartography

Analytical cartography differs greatly from the other approaches to the main field of cartography. The communication school of cartography, founded by Arthur Robinson (1952) with his provocative dissertation, sees the underlying operational concept of cartography as communicating spatial information graphically to the reader via a map. Robinson's work sparked a blooming of spatial cognition research in cartography during the 1960s and 1970s, culminating with the book by Robinson and Petchenik on the *Nature of Maps* (1976).

A much older approach to cartography that has been a going concern since 2500 B.C. with Babylonian cuneiform clay tablets is the notion of cartography as map production. This approach has continued since classical times strengthened with works such as Ptolemy's book *Geographia* in about 150 A.D., which, 1350 years later helped to spark the European exploration of the world, generating the "golden age of cartography" from about 1500 to 1700, when the findings from these explorations were processed and compiled into new and exciting maps. This tradition of cartography as map production continues to the present day as a host of regular series of conventional topographic and other kinds of maps and charts. Also included are exciting new computer technologies developed to design and create interactive cartographic systems, animations of spatial data, and digital spatial databases (Williams 1996). MacEachren (1995) has articulated a more contemporary view of this activity with maps as "spatial representation" that takes "a combined cognitive-semiotic approach to maps and mapping." The recent U.S. National Report to the International Cartographic Association Congress in Ottawa (Brewer 1999) shows the wealth of work going on in the broader field of cartography.

Analytical Cartography and Relations to Cognate Sciences

Analytical cartography has continued to develop and mature during the last few decades. Nyerges (1980) sees analytical cartography as the intersection of the fields of cartography, discrete mathematics, geography, computer science, and image analysis. This is a clear articulation of the relations of analytical cartography to other fields as evidenced by the research work of Tobler and others who have followed him in the field. Kimerling (1989) recognized analytical cartography as a major sub-field in cartography in his chapter in the premier volume on *Geography in America*, listing topics such as cartographic data models, information structures, cartographic data quality, cartographic data transfer standards, map projections, spatial data interpolation, analytical map generalization, and numerical map analysis as being part of this sub-field. These topics clearly relate to the four areas identified by Nyerges above.

Analytical cartography has a considerable overlap with geographic information science (GISci), with the primary differences arising in how the base of conceptual theory is directed and used. Marble (1979) recognized that there is about an 80 percent overlap between the theoretical base of spatial theory from which the two areas draw. Moellering (1991b) largely agreed. In the 1990s, this situation was recognized by researchers in the area of GIS, and leaders such as Goodchild (1990) began to call for an expansion of the theoretical base of the field in what he called "spatial information science." Moellering recognized the relationship between analytical cartography and GISci as close and symbiotic.

However, there are also many relationships and similarities between analytical cartography and other similar fields in this area. Bergougnoux (1997) in his editor's opening statement in the premier issue of *GeoInformatica* articulates the need to bridge the gap between computer science and geography by expanding the theoretical basis of this area of research of geographic information science. Miller (1999) in his discussion of the relationship between spatial analysis and geographic information systems for transportation (GIS-T) identifies several areas of research in analytical cartography germane to the problem. Recently, Getis (2000) edited a dedicated issue of the *Journal of Geographical Systems*, with many authors, that explores various relationships between spatial analysis and GIS. Many relationships between these two areas and analytical cartography can be recognized. One can also identify conceptual relationships between geostatistics (Cressie 1993) and analytical cartography.

Armstrong (2000) seeks expanded relations between geography and computational science because he sees "many spatial analysis problems are compute-bound problems." Marble (2000) advocates further increasing the facility with modern computing concepts and techniques between spatial analysis and GIS, especially with object-oriented approaches. The limitations of expanded computer computational power as articulated by Saalfeld (2000) are perhaps more acute in GIS because much of the work there is generally more empirically based. Frank (2000) sees new directions for geographic information science in the use of multi-agent systems, field computing and virtual reality, object-orientation and category theory.

Fundamental Conceptual Theory in Analytical Cartography

Tobler's 1961 dissertation on *Map Transformations of Geographic Space* can be recognized as the beginning of analytical cartography. From this initial point Tobler has developed many conceptual parts of analytical cartography, and his work has inspired many others to expand the conceptual base of the field. The following is an exploration of some of the fundamental concepts in analytical cartography.

Geographic Map Transformations

In his novel and creative research work for the dissertation, Tobler systematically explored the concept of spatial coordinate transformations, showing some of the ways in which such transformations can be used. He proved that map projections are a special case of spatial coordinate transformations. He also explored ways to compare the similarity of map projections (Tobler 1965b; 1986b).

Tobler (1965a) also carried out research to analyze the correspondence of spatial patterns and later developed an approach to the comparison of spatial outlines utilizing spatial regression techniques based on pairs of homologous landmark points (Tobler 1978; 1994). One additional outcome is the mathematical development of cartograms (Tobler 1973; 1979b; 1986a), where he is the leader in the field. Later Tobler (1979a) extended his ideas of cartographic transformations to conceptually explain the notion of the cartographic progress as practised by most conventional cartographers at that time.

More recently, several researchers have sought to explore these and related transformations. Clarke (1990) explored several forms of map and data structure transformations; Lin (1998) examined the variety of algebraic geometric data transformations; Palma and Benedetti (1998) advocated using transformations with linear operators that may produce maps with a different topology; and Chrisman (1999) proposed an approach to map transformations organized primarily by their geometric and attribute characteristics.

Real and Virtual Maps

In the early 1970s, Morrison recognized that the conventional definition of the map was clearly insufficient in light of the newer technological and computer developments in the field. In his article in the premier issue of the *American Cartographer*, Morrison (1974) called for an expanded definition of what was at that time considered to be a map. Moellering took up Morrison's challenge by developing the concept of "real" and "virtual" maps (Moellering 1977b; 1980a; 1984). This expanded concept of the fundamental nature of a map extended Tobler's original concept of map transformations to a new level, while responding to Morrison's challenge for a new definition of a map that encompassed all of the new kinds of cartographic products being developed. In a nutshell, the concept of real and virtual maps is based on two crucial characteristics of maps and spatial data:

- Whether a map is directly viewable as a cartographic image; and
- Whether it has a permanent tangible reality.

These two critical characteristics are then cross-classified into a four-class concept of real and virtual maps. They are defined as follows from Moellering (1980a):

- *Real Map*—Any cartographic product which has a directly viewable cartographic image and has a permanent tangible reality (hard copy). There is no difference as to whether that real map was produced by mechanical, electronic, or manual means.
- *Virtual Map, Type I*—Has a directly viewable cartographic image but only a transient reality as has a CRT map image. This is what Riffe called a temporary map. Given the direction of current scientific work, electro-cognitive displays may be possible.
- *Virtual Map, Type II*—Has a permanent tangible reality but cannot be directly viewed as a cartographic image. These are all hard copy media, but in all cases these products must be further processed to be made viewable. It is interesting to note that the film animation adds a temporal dimension to the cartographic information.
- *Virtual Map, Type III*—Has neither of the characteristics of the earlier classes but can be converted into a real map as readily as the other two classes of virtual maps. Computer-based information in this form is usually very easily manipulated.

These four classes of real and virtual maps are exhaustive classes, so that all kinds of existing maps and map products fit into this classified system. One can also note that all new cartographic products invented in the future will also be covered by these classes of real and virtual maps because they are exhaustive.

One can then extend Tobler's concept of cartographic transformations in a very real way to realize 16 real/virtual possible map transformations as discussed in Moellering (1980a; 1984). The following eight examples can be easily recognized:

- Real => Real – conventional cartographic processing;
- Real => Virtual 3 – digitizing spatial data and storing them in a digital database;
- Virtual 1 => Real – making hard copy image of a CRT screen image;
- Virtual 3 => Real – digital cartographic plotting/drawing from a spatial database;
- Virtual 3 => Virtual 1 – CRT display of digital spatial data from hard disk to CRT;
- Virtual 1 => Virtual 3 – CRT screen editing spatial data stored on hard disk;
- Virtual 2 => Virtual 3 – reading digital data from CD-ROM and storing them on hard disk;
- Virtual 3 => Virtual 3 – mathematical transformation of digital spatial data resident on computer magnetic media.These transformations are sometimes called Tobler's transformations.

The other eight transformations can be defined (Moellering 1984), and many have practical uses. Together, these 16 real and virtual map transformations define *all* of the cartographic and spatial data processing steps that exist in cartography, and the spatial data sciences. In fact, Virtual 1 and Virtual 3 transformations are the key to the understanding of the operation and running cartographic data processing systems (Moellering 1984).

Real and virtual map transformations have been used to specify the logical structure of spatial data systems. Moellering (1983; 1984) described the use of such transformations in the design of an interactive spatial data analysis and display system. At least seven of the 16 real/virtual map transformations are used for this logical design. The Virtual 1 <==> Virtual 3, Virtual 1 <==> Virtual 1, and Virtual 3 <==> Virtual 3 transformations dominate the logical architecture of the system. Hence, almost all of the processing inside the system is being done at the Virtual 1 and Virtual 3 levels of operation the lower half of the real/virtual map conceptual domain, which is not hard copy. This shows a major reason why such systems dominate the spatial data sciences, because they operate in the half of the domain where computational and graphic operations are much more fluid and manipulable.

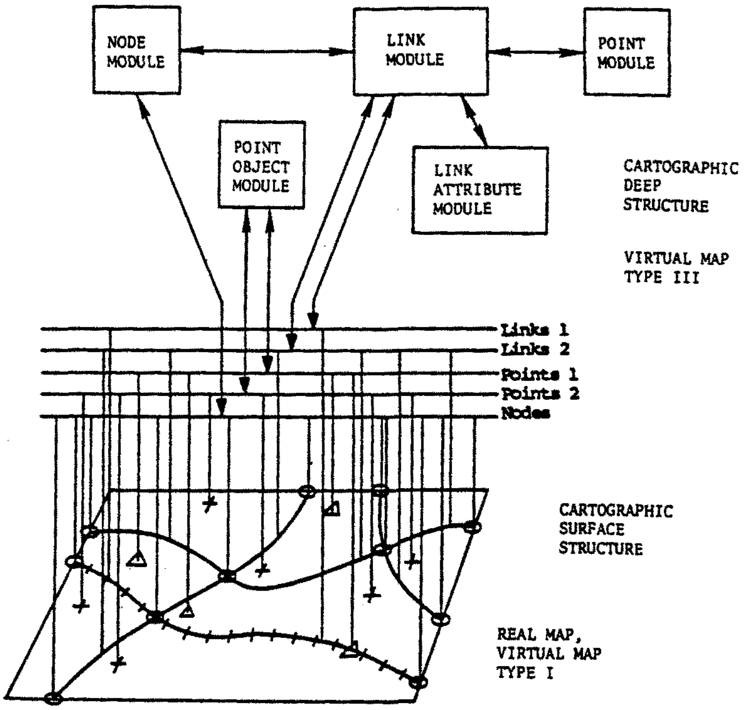

Figure 1. Deep and surface cartographic structure. Deep structure is associated primarily with Virtual 3 maps, while surface structure is associated primarily with real and Virtual 1 maps.

Moellering (1983) has shown that these 16 Real and Virtual map transformations can be used for the logical design of a spatial data analysis and display system. Naturally, the use of Real and Virtual maps transformations is used for interactive spatial data systems design in conjunction with other standard kinds of system and software design tools.

Deep and Surface Structure in Cartography

A second major concept in analytical cartography is Nyerges' deep and surface structure (Nyerges 1980). Here Nyerges took Chomsky's (1965) basic structural linguistic syntactical relationships, adapted them over to cover spatial data relationships, and then related them to real and virtual maps as shown in Figure 1.

Here surface structure in a cartographic setting becomes cartographically displayed spatial data (Real or Virtual 1 form), while deep structure becomes spatial data and relationships stored in a non-graphic (usually digital) Virtual 3 form on some kind of magnetic medium. Now one can see cartography in a new conceptual light and realize that 99 percent of cartographic work, since the Babylonians, has dealt with cartographic surface structure. Analytical cartography deals primarily with spatial and cartographic information in the deep structure domain, usually in a Virtual 3 form. Hence, we can now realize that analytical cartography dealing with deep cartographic structure is the "new half" of the field that has been operating since its founding by Tobler in the early 1960s. One now can see much more clearly the research opportunities offered by analytical cartography in terms of its conceptual definitions in deep/surface spatial structure, and in terms of real and virtual maps.

More recently, Nyerges (1991a) in his analysis of analytical map use, examined the content and structure of virtual 3 maps in the deep structure and how they related to the information content of virtual maps. He then related this to the entity class/type and characteristic data domains of spatial data, and various data model levels that will be discussed in the following section. Nyerges (1991b) used the concept of the "meaning triangle" to help the pair-wise transformations between surface and deep structure to illuminate relationships between spatial referents in the real world, symbols that represent these referents, and concepts of meaning that are shared. This work adds further conceptual structure to deep and surface structure and our understanding of the spatial world in which we live.

Nyerges Data Levels

A third fundamental concept, developed by Nyerges (1980), is the notion of cartographic data levels. He saw that when one is dealing with spatial data, that these exist at several levels of abstraction, now known as *Nyerges data levels*. Nyerges clarified what up until then was a very confusing situation in cartographic data, and in the spatial data sciences in general. This work has resulted in a six-level definition of cartographic data structures defined by Nyerges (1980) as follows:

- *Data reality* – The data existing as ideas about geographical entities and their relationships which knowledgeable persons would communicate with each other using any medium for communication.
- *Information structure* – A formal model that specifies the information organization of a particular phenomenon. This structure acts as a skeleton to the canonical structure and includes entity sets plus the types of relationships which exist between those entity sets.

- *Canonical structure* – A model of data which represents the inherent structure of the data and, hence, is independent of individual applications of the data and also of the software or hardware mechanisms which are employed in representing and using the data.
- *Data structure* – A description elucidating the logical structure of data accessibility in the canonical structure. There are access paths which are dependent on explicit links... Those access paths dependent on links would be based on tree or plex structures such as network models. Those access paths independent of links would be based on tables as in relational models.
- *Storage structure* – An explicit statement of the nature of links expressed in terms of diagrams which represent cells, linked and contiguous lists, levels of storage medium, etc. It includes indexing how stored fields are represented and in what physical sequence the records are [sic] stored.
- *Machine encoding* – A machine representation of data including a specification of addressing (absolute, relative or symbolic), data compression and machine code.

Although other general spatial data models with fewer levels have been discussed in the literature (Peuquet 1984), Nyerges' six-level data model is still the best for both conceptual understanding ,and for pedagogic purposes, because it provides for explicit specification of data levels as a spatial information system carries out its real/virtual map transformations.

Further work by Nyerges (1991a; 1991b) on geographic information abstractions helps to illuminate the relationships between various data levels, and deep and surface structure. Here he is working primarily at the information and canonical structure levels and how they are used to abstract spatial information from the real world. Nyerges' use of the "meaning triangle" helps to elucidate entity definitions and associated properties of space, time, and theme, and the proceeds to develop a partial conceptual database design at the information and canonical structure levels.

Spatial Primitive Objects

In order to store digital encodings of spatial objects that are digital representations of real-world entities, these fundamental spatial primitives and objects must be defined and specified. These primitive and simple objects serve as fundamental digital building blocks of spatial data structures in Virtual 3 spatial databases. Although the basic spatial dimensionalities have been sorted out relatively early by Bunge (1962) and Nystuen (1963), the definition and specifications of digital primitive and simple objects remained illusive. In the U.S., systematic work to specify a unified set of spatial object primitives began in earnest with the founding of the U.S. National Committee for Digital Cartographic Data Standards in 1982 (Moellering 1982), which, after five years of detailed scientific work, developed a full set of 0-, 1-, 2-D spatial primitive and simple objects that became the conceptual heart and soul of the American Spatial Data Transfer Standard (NCDCDS 1988), now known as the Federal Information Processing Standard (FIPS) 173-1. These 0-, 1-, and 2-D spatial primitive and simple objects can be used to construct almost any kind of digital spatial data objects from zero through three dimensions. Clarke (1990) provided implementations of some of these spatial objects in the C language that have been widely adopted by some in the software area of the spatial data sciences. However, more work is needed on the more exotic 3-D primitives and the relative topologies associated with them.

The Sampling Theorem

One of the simplest, basic, and most important spatial theories is the sampling theorem. This theorem specifies desired sampling intervals in one and two dimensions for sampling along a linear object (such as a line) or an area object (such as a surface). The equation:

$$\Delta = \tfrac{1}{2}\lambda$$

where:
Δ = resulting theoretical sampling interval; and
λ = the finest assumed spatial wavelength in the 1-D or 2-D object being sampled.

This seemingly simple equation is the theoretical key to many questions of resolution and spatial sampling. Proper use of this fundamental piece of spatial cartographic theory facilitates the proper use of sampling, interpolation, neighborhood operators, pixel resolution, and a host of other applications. The sampling theorem was originally borrowed from mathematics, and the above equation is intended for use in a regular segment or cellular setting. Later, Tobler (1984) showed how average resolution could be approximated in an irregular polygonal domain with the concept of resels. In his most recent work, Tobler (2000, this issue) has generalized this estimate for resolution in the irregular domain to N dimensions as shown on next page:

$$\text{Average Spatial Resolution (ARS)} = \left(\frac{Km^d}{N}\right)^{1/d}$$

where d is the spatial dimension of the region being analyzed; Km is the total number of kilometers to the d^{th} power of units in the mesh; and ASR is d^{th} root of that total number of units. (See the appendix to Tobler's article (2000, pp. 189-194) for details.

Every real world spatial entity exists in both space and time, with time being usually recorded as a single fixed temporal point. According to Peuquet (1999, personal communication), time moves through the time/space continuum. Research work in recent decades has recognized that time is a continuum, and this work has tried to develop various strategies to record time in one dimension, in a manner analogous to the way we treat space in two or three dimensions. Early workers in dynamic cartography (Tobler 1970; Moellering 1972; 1973a) recognized this fundamental temporal characteristic of spatial data when they created their spatio-temporal (Moellering 1973a; 1976; 1980b) computer animated displays.

However, a special animated spatio-temporal display is different from designing and building a Virtual 3 spatial database system that records and handles more than one point in time for specific entities encoded in that spatial database. Peuquet (1999) has researched temporal aspects of spatial data systems and is raising this challenge to the conceptual community. Most workers conceive of time as a linear thing, but Moellering (1973a) reported that one can treat time as a circular variable when analyzing such spatial variables as traffic crashes. In that piece of research in analytical visualization time was treated as a circular variable based on the day of the week. The result of this time transformation was an extremely effective spatio-temporal display (Moellering 1973b; 1976). Other such temporal cycles are theoretically possible.

A Selection of Analytical Theory in the Field

It is now possible for one to identify a host of more specific pieces of mathematical spatial theory flow from the fundamental theoretical concepts presented and discussed above. Many of these pieces of spatial theory were originally developed in mathematics, statistics, or other systematic sciences and have been brought over and adapted for use in analytical cartography and other spatial data sciences. In many cases this theory has been considerably extended to fill a conceptual need in the body of theory in analytical cartography. In the following section an effort will be made to touch on the more important pieces of spatial analytical theory, while other items of lesser importance must be passed over. It should be recognized that much of this early adaptation and extension work in analytical cartography has been done by Tobler himself. (See Tobler's (2000, pp. 189-194) and Clarke and Cloud's (2000, pp. 195-204) papers in this issue which provide some clues to this adaptation and extension process.)

The View of Spatial Frequencies

One of the most general concepts involves how one conceptualizes a spatial 3-D surface. Because of the existence of the sampling theorem, spatial surfaces can be viewed as spatial frequencies:

$$Z = f(X,Y)$$

where:
X,Y provide the location; and
Z = height above the datum.

Spatial surfaces can be conceived of being made up of many different spatial frequencies, each of which has its particular wavelength, magnitude, and orientation (see, for example, Rayner 1971.) Added together, any spatial surface can be composed of perhaps hundreds of spatial frequencies. Each particular frequency contributes its part to the overall mathematical definition of the surface. Hence, any operation carried out on the surface—regardless of whether it is quantitative, analytical, or visual—has some effect on the surface by altering the spatial frequencies contained in it. Spherical harmonics may also be considered. This conceptual and theoretical view of spatial surfaces then serves as a fundamental conceptual foundation for many of the pieces of spatial theory that are discussed in the following sections.

Spatial Neighborhood Operators

Spatial neighborhood operators can be designed to operate on any kind of regular or irregular, topologically well behaved set of polygons. Tobler brought the concept for the regular cellular domain in from Holloway (1961), the seminal reference to quantitative neighborhood operators for the early researchers in the field. Holloway showed clearly the relationship between the data domain and the spatial frequency domain, and how neighborhood filters could be designed and used on spatial surfaces. Although Holloway's work focused on regular cellular structures, both square and hexagonal, his work proved to be singularly instructive to the field as articulated by Tobler (1967, 1969b) when the concept was presented to the community of analytical cartography. Rosenfeld and Kak (1982), Young and Fu (1986), Maître and Zinn-Justin (1996) and other researchers of a similar ilk have generated a wide array of work on neighborhood operators that is useful for image processing and other tasks. This research, and that by Wang et.al. (1983) and Brownrigg (1984), has shown that square-cell neighborhood operators can be used for smoothing, edge enhancement, and other spatial operations. The effects of quantitative neighborhood weight field operators can now be understood in terms of the theory of spatial frequencies. More recently, the notion of wavelets has entered use as a neighborhood filter for edge detection (Farrier et al. 1995) and multi-resolution terrain generation (McArthur et al. 2000). However, a big limitation on Holloway's concept is that it does not work in the domain of irregular polygons. Guptill (1978) showed how such neighborhood filters in the square cell domain could be designed and used on nominally scaled data using a weighted probability approach for the neighborhood operator.

Spatial Adaptations of Fourier Theory

Fourier theory is a mathematically defined piece of theory that can calculate the wavelength and magnitude of spatial frequencies in a regularly defined, cellular surface of Z values. It is tied to the sampling theorem in the square cell domain in terms of how it calculates, resolves and approximates the spatial frequencies, wavelengths, magnitudes, and orientations in a square cell spatial surface. If a given candidate surface meets the cellular, mathematical, and continuity assumptions required, then the researcher can calculate the spatial frequencies, magnitudes and many other spectral components resident in a spatial surface. Both Tobler (1967) and Rayner (1971) were early researchers in this area, adapting much of Fourier's mathematical theory to analytical cartography and the spatial data sciences. Tobler (1969a) introduced the novel idea of calculating a linear spectrum along the route of U.S. highway 40, associating some of the frequency variations with variations that could be due to variations in Christaller's central place theory. Few seem to have followed up on this innovative and provocative scientific idea.

More recently, Dougherty and Moellering (1996) have shown how Fourier analysis can be used to calculate signatures for numerical terrain types. However, the problem remains that, generally, Fourier frequencies and magnitudes cannot be calculated in an irregular polygonal cellular approximation of a surface. One exception to this problem is the work of Moellering and Tobler (1972), where the relative average magnitudes were approximated as increments of a variance spectrum in a hierarchically defined system of irregular polygons, which has an *a priori* fixed hierarchical structure that meets some additional spatial constraints.

Spatial Analytical Uses of Information Theory

Information theory holds much promise for analytical cartography. The concept is usually employed as an entropy measure of some spatial distribution. Entropy is usually reckoned for a parameter H of a spatial distribution that is being optimized. The great theoretical advantage of entropy measures is that they are distribution-free statistics, whereas most least squares measures, such as those used in geography, surveying and geodesy, were assumed to have a Gaussian parametric distribution. Many spatial, statistical quantitative Z distributions do not have a normal Gaussian parametric distribution, and if not corrected, can severely bias the results of the analysis. Entropy statistics are distribution free, i.e., there is no underlying assumed base for the statistical distribution. Hence, the resulting entropy statistic, usually H, cannot be biased by a non-Gaussian or other strange underlying statistical Z distribution. Wasilenko and Moellering (1977) showed how entropy statistics can be used to calculate better choropleth map classes. The entropy statistic has also been shown useful in approximating a range of data classes suitable for a particular Z distribution. Other kinds of entropy models have been used in analytical cartography.

Fractal Spatial Operators

Fractional dimensions, known as fractals, play an important role in analytical cartography. Fractals are a spatial adaptation of the mathematical definition of the Hausdorff-Besicovitch measure of spatial dimension (Mandelbrot 1977). Most cartographers and spatial scientists conceive of things in the real world as having integer 0-, 1-, 2-, 3-D, etc. fixed physical dimensions. Hausdorff-Besicovitch have shown that spatial phenomena in the linear and areal domains can be conceived of having dimensionalities that are decimal numbers. Spatial work with fractals was initiated by Mandelbrot in his paper titled "How Long is the Coastline of Britain?" (1967); he drew on earlier work by Richardson (1961). This insight stimulated several researchers to become interested in the potential for fractal theory in analytical cartography. Goodchild (1980) investigated the use of fractals in the analysis of physiographic surfaces. Shortly thereafter Shelberg et al. (1982) presented what has become known as the "Shelberg algorithm," which can be used to determine the fractional dimension of lines analytically. Quie (1988) utilized this theory on the coastline of Louisiana, while Clarke and Schweitzer (1991) developed a more robust fractal estimator for natural terrain. A comprehensive book on the spatial use of fractals was compiled by Lam and DeCola (1993). Outcalt et al. (1994) analyzed landscape physiography using fractal theory. Fractals can be used to analyze and measure both regular and irregularly partitioned 1-D lines and 2 1/2-D surfaces. Dutton (1981) has shown that fractals can also be used to enhance cartographic line detail, while Goodchild and Mark (1987) have shown that certain types of fractal surfaces could be used as default surfaces against which certain terrain processes could be assessed. Most recently, Duckham and Drummond (2000) used fractals to assess error in digital vector data.

Critical Features and Warntz Networks

One of the more interesting ways of analyzing surfaces is to determine the mathematically critical points and lines on the surface where the first and second spatial derivatives change. Warntz (1966) defined these as peaks and pits, ridges and valleys, and passes and pales, which can be collected in a topologically systematic way to construct the topological skeleton of the surface. The topological skeleton of a surface is formally called the Warntz Network, after William Warntz (1966), who proposed the concept.

Warntz Networks provide those in analytical cartography with a powerful tool to estimate the visibility, line-of-sight and slope/gradient of surfaces. Peucker and Douglas (1975) and Toriwaki and Fukumura (1978) developed and tested pass location algorithms to assist the building of Warntz Networks, while Bennett and Armstrong (1996) have taken a somewhat broader approach to this kind of analysis. Wilcox and Moellering (1995) presented the Wilcox algorithm that more effectively assists in the building of Warntz Networks. This notwithstanding, it should be noted that the basic topological theory still appears to have some gaps in it, and needs more deeply theoretical work before such approaches are complete. There is also a fairly large literature in hydrology (see, for example, Band 1986).

Polygon Analysis, Overlay and Transformations

Polygon analysis is a very broad and wide-ranging area of research in the field. Most of the early work in the spatial sciences on irregular cellular data was accomplished by interpolating back to regular, square cellular systems where the standard mathematical theory usually worked. Lam (1983) provides an interesting review of the work done until the early 1980s. However in recent years, research was carried out to extend the theory from regular cellular analysis into the irregular domain. This thrust has resulted in the development of some spatial neighborhood operations that work in the irregular domain. Early work by Tobler (1973; 1984), Chen (1986), Rhynsburger (1972) and Gold (1999) on Thiessen and Delunay polygons helped to extend this effort. Wagner (1988) analyzed the polygon overlay process and found that there are at least 34 different cases of how irregular 2-D polygons could overlay each other. This finding lead to the realization that several of the leading polygon overlay algorithms (Goodchild 1979; Miyashita et al. 1985) were theoretically insufficient because they did not take into account all of the topologically possible cases of polygon overlay. This could produce errors in output and empirical results under certain conditions.

Van Roessel (1991) developed a plane-sweep algorithm that seems to avoid the problems discussed above. Saalfeld (1991) provided a neat analytical solution from the point of view of algebraic topology, and Franklin (2000) addressed some of the problems of implementing polygon overlay processes. The polygon analysis and overlay approaches discussed above work with polygons in a two- dimensional setting. Tobler (1979c) proposed an elegant mathematical approach to the transformation of polygonal structures into a smooth pycnophylactic (volume-preserving) surface. This theoretical concept was later extended to polygons on a sphere (Tobler 1996).

Map Generalization

The notion of map generalization has always been a part of the map creation and production process and, as such, it has existed in cartography for centuries. In recent decades, more systematic graphic approaches were developed in the form of graphic rules for the production of real maps. With the development of digital computer systems in the last several decades, more work has been devoted to developing numerical approaches to map generalization. It should be said that this is a very complex cartographic problem worthy of the attention of some of the best minds in the field. Most of the work has so far has resulted in the development of numerical algorithms and some numerical measures which primarily reside in the surface structure, and, sometimes, have a rather shallow base in the theory. This is primarily the case because most of the work in cartographic generalization has, taken place in the surface structure realizations of the cartographic imagery, rather than down in the deep structure where most of the mathematical analytical theory probably resides.

Several numerical algorithms have been developed that are widely utilized, such as that by Douglas and Peucker (1973). Peucker's (1976) paper describes the underlying concepts based on geometric surface structure properties. Later work by De Berg et al. (1998) attempted to implement this bandwidth approach to line generalization while maintaining a topologically correct local neighborhood. These sorts of strategies of line simplification were evaluated from the point of view of multi-scale representation by Barber et al. (1995). Further work by Dettori and Puppo (1996) has begun to specify the ways in which generalization interacts with the geometry and topology of a map structure. As time goes on, a more sophisticate and theoretical approach is slowly developing for this very complex and vexing problem. Brassel and Weibel (1988) have begun to specify a more conceptual base for the scientific enterprise of map generalization, although much of it is still in the surface structure domain. In the 1990s, a wealth of research was published that relates to cartographic generalization, including a monograph by McMaster and Shea (1992) and a special content issue of *Cartography and Geographic Information Systems* edited by Weibel (1995).

More recently, Richardson and Thomson's (1996) have discussed an approach to road network generalization that involves bringing together thematic, geometric and topological information. Weibel (1992) has begun to make some real theoretical progress with the generalization of terrain surfaces by building an approximation to the local Warntz Network of the surface, and then making analytical modifications to it and the resulting skeletal structure of the surface. This approach shows some real analytical promise for deep structure analysis. Müller et al. (1995b) reviewed the state of the art in cartographic generalization, and have provided some interesting insights. One insight that is evident from their discussion is that more progress is possible if deeper levels of analytical theory are used to address the problem. This work is the first chapter in a book by Müller et al. (1995a) on generalization that illustrates the general state of the art in the field. Terei (1999) takes a comprehensive view of adaptive filtering. It is clear that progress is being made on this problem that has challenged cartographers for centuries, and that more analytical approaches, many in the deep structure, are helping this effort.

Shape Analysis

Shape analysis has always been an illusive analytical challenge to the spatial sciences. Early work in the field of geography to develop single parameter measures of shape, usually the compactness characteristic, was theoretically deficient because the units of measure did not cancel. Good measures are dimensionless and have a numerical range from 0.0 to 1.0 (and sometimes -1.0). Moellering and Rayner (1979) reviewed these measures and found most of them wanting. They then went ahead and developed, in the mathematically robust, complex domain, an analytical measure of shape based on Fourier analysis (Moellering and Rayner 1980; 1982). Called the Dual Axis Fourier Shape Analysis (DAFSA), this measure is independent of location, orientation, scale, number of points, and the beginning of the string of points in the outline. With this DAFSA analytical approach to shape, they showed how to calculate a variance spectrum of any topologically well behaved, closed shape.These frequencies and magnitudes could then be used to interpret the mix of spatial frequencies that make up the outline of the shape, this time in the complex 2-D domain.

Shortly after publishing the analytical DAFSA, Moellering and Rayner (1984) developed an analytical approach to measure the shape correlation between two closed outlines. This measure, known as DAFSA harmonic correlation and coefficient of determination, is analogous to the Pearson correlation coefficient and coefficient of determination in statistics. The DAFSA shape correlation coefficients range from 0.0 to 1.0 and are interpreted in the same way that the Personian coefficients are interpreted. More recently, Clementini and Felice (1997) have examined the fundamental aspects of shape in terms of topological, projective, and metric properties, and Kunii et al. (1997) have examined diverse questions in shape modeling.

Spatial Data Models and Structures

Spatial data structures have existed from the earliest days of digital cartography. In those days, spatial data structures were geometry that only utilized implicit topological relationships at best. During this time there were many debates about the need to develop topological data structures, and the term "spaghetti file" was coined to dramatize the shortcomings of geometry-only spatial data structures (Chrisman 1972). Many data structures that were designed went directly from the real-world entities (Nyerges Level 1) to the data structure (Nyerges Level 4), bypassing the information structure and canonical structure levels (Nyerges Levels 3 and 4). As a result, many early spatial data structures, especially geometry-only structures were rather ineffective, or in some cases failed outright.

It came to be realized that in order to be effective, topological concepts had to be incorporated into spatial data structures. Corbett (1979) introduced into cartography data structures the notion of including explicit topological properties and characteristics into the overall design of the structures. One of the first such structures as the GBF/DIME (Dual Independent Map Encoding) structure developed by the U.S. Bureau of the Census in the late 1960s. Not long after, the wider discussion on the nature and design of topological data structures was initiated by Peucker and Chrisman (1975). About that time, a conference on spatial data structures at Harvard (Dutton 1978) brought together a large number of researchers specializing in this area, which produced interesting results.

Even then, the notion of clear data levels, as later refined by Nyerges (1980) was not widely recognized or understood in the specialization. Nyerges (1981; 1991b) indeed did clarify this situation with his development of a six-level model for spatial data, discussed above. About the same time, Peuquet (1981) examined the host of different spatial data structures that were evolving. Although Peuquet and Nyerges did not agree on the number of conceptual levels that were most appropriate in the theoretical model of spatial data levels, this discussion was healthy and served to promote scientific growth in the field. Not long after, a very elaborate planar spatial data structure called TIGER (Topologically Integrated Geographic Encoding and Referenced) was developed by the U.S. Census Bureau (Marx and Broome 1985; Broome and Meixler 1990).

It was also realized that more than just 2-D planar models were needed for terrain and bathymetric surfaces. Peucker and Chrisman (1975) introduced the idea of the Triangulated Irregular Network (TIN) spatial data model to the field. The notion of a TIN model was further elucidated by Peucker and his students (1978), which showed the efficiency of the TIN model under certain conditions. This topic was much more fully investigated by Kumler (1994) who compared the relative efficiencies of TIN and DEM terrain models. Such comparisons for operational flexibility and efficiency were also carried out by Abdelguerfi et al. (1998) and by Franklin and Gousie (1999). Some of the associated implementation problems are discussed by Franklin (2000).

Research in global tessellation models has been taking place in recent years with the effort of Dutton (1988) and White et al. (1992). Further work by White, et al. (1998) has progressed to the examination of various kinds of areal and angular distortions on tessellation partitionings on the sphere. Dutton (1999) provides a general summary of the situation.

The recognition of many of the concepts that underlie the design of a spatial database was neatly summarized by Nyerges (1981) with an articulation of his six data levels (as discussed above) and continued with the discussion of some of the syntactical, semantic, lexical and graphic issues involved in spatial data models. Interest in spatial data models has extended to GIS and many other cognate areas in the spatial data sciences. Frank (1991) focused on the design of cartographic databases, while Salgé (1991) discussed the development issues involved. In a recent book, Molenaar (1998) brings together much of the research done so far on cartographic databases. Likewise Richardson assembled a set of papers that document the incorporation of spatial, semantic, and temporal data in spatial data structures (1996a), providing an example of how to build a spatial database and generate direct abstractions (Richardson, 1996b). The problems associated with amalgamating spatial databases while maintaining topological continuity in a setting of seamless spatial data interoperability are examined by Laurini (1998). Further development is continuing in this area; some of this work is moving towards object-oriented data modeling.

Analytical Visualization

Most people conceive of visualization as the spatial representation of spatial data (MacEachren 1995). That is largely true for most of conventional hard copy cartography that is concerned with real maps, where the goal of the map is to convey the data to the reader in an efficient way. However, there is an approach to visualization that one might characterize as analytical visualization (AV), and what some people might view as a scientific oxymoron. The differentiation is one between trying to represent the data (or communicate them to the reader) versus the idea of visualizing them as an integral part of a process to analyze spatial problems, or as part of the result of the analysis. Although this definition is not precise, it will be used as the basis for this discussion. Much of what MacEachren (1995, pp. 355 ff.) characterizes as Geographical Visualization (GVIS) falls into analytical visualization. Analytical visualization is distinguished from Virtual Reality Model Language (VRML) displays in computer graphics (Nadeau 1999) in that the AV includes spatial visualizations that are the result of some analytical/mathematical process or are used to analyze such a process. Clearly, VRML includes many non-geographic/cartographic types of displays, or those which are straightforwardly representational. The taxonomy of computer visualization systems by Roman and Cox (1993) may be helpful .

In analytical visualization, technology from the surface structure is combined with some sort of analytical concepts or theory from the deep structure, which results in a visualization process that is an advance over what existed before. The seminal article by Horn (1982) describes the progress in visualization since the early days in both cartography and in computer graphics, a unique appraisal of two sets of literature from two separate disciplines. It turns out that one of the earliest pieces of research on analytical visualization is by Wiechel (1878), who provides the mathematical equations for calculating gray-level reflectance on spatial surfaces at a point. Much later, work by Brassel (1974) continued this line of work on gray-level relief shading of terrain surfaces, which even began to develop a number of equations that took into account atmospheric variations in mountain areas. That work, and other work like it, combined with terrain visualization work in computer graphics resulted in Horn's comprehensive summary. More recently, Moellering (1989b) combined gray-level relief shading with overlays of thematic satellite imagery in an image where both scenes could be seen simultaneously co-registered together. Pike and his colleagues at USGS (Thelin and Pike 1991; Reichenbach et al. 1993) moved towards much larger gray-level relief images which are very striking, especially their full-size 6,046 x 3,750 image of the United States. Kirschenbauer and Buchroithner (1999) added holography to the discussion.

Moellering and Kimerling (1990), and Moellering (1993), developed an analytical approach to terrain illumination, utilizing color hues that preserve the perceptual relief effect and eliminate the well known directional bias of gray shading. For their groundbreaking work they were awarded two patents for a process known as the MKS-ASPECT™ SLOPE-ASPECT COLOR SURFACE RENDERING PROCESS. Brewer (1997) presented work that endorses the use of various spectral hue sequences with quantitative spatial data and recommends further research into the topic. Additional analytical visualization opportunities such as these are possible.

Beyond static and terrain displays lie interactive and dynamic virtual visualization environments. Interactive cartographic environments have been recognized by Moellering (1977a) as a very rich information channel that offers much potential for cartographic visualization, as reiterated by Andrienko and Andrienko (1999). A host of analytical interactive spatial visualization systems have been developed over the years. They have ranged from systems that use an analytical method to process and display the data (Moellering et al. 1977)—such as transportation route design—to systems specifically designed to investigate various kinds of data relationships between spatial variables, such as the work by Monmonier (1989) on the Exploratory Data Analysis (EDA). Eich (2000) provides some additional interesting ideas.

Spatial visualization has continued to expand with the development of a wide range of visualization hardware and technology. The spatial data sciences have actively participated in this technical growth and development. MacEachren and Kraak's (1997) special issue of *Computer and GeoSciences* on "Exploratory Cartographic Visualization" provides several examples of this participation via analytical visualization. MacEachren et al. (1999) published the results of their work on developing Knowledge Discovery Databases (KDD) from the spatio-temporal data of many variables in 1999, providing a new means of utilizing spatial knowledge databases linked via spatial visualization.

Enhancing the analysis and display functions of these approaches is discussed by Brooks (1999). Brooks argues that virtual reality has progressed from an experimental technology in the time of Ivan Sutherland in the 1960s, to the present where it is a mature visualization tool that can be utilized in many disciplines. Moellering (1989a; 1990a; 1990b; 1991d), who was an early cartographic worker in true 3-D stereoscopic vision, agrees with this assessment, as do Kraak (1994) and Verbree et al. (1999). There are many possibilities for using interactive analytical visualization in cartography and the spatial data sciences. One of them is for systems analysis (Parsons and Wand 1997). An additional means to extend interactive analytical visualization is the World Wide Web (WWW) as discussed by Cartwright (1999).

Dynamic cartography, or cartographic animation, existed before computers became widespread. However, the hand-made, hard copy Virtual 2 cartographic products were almost always representational, and hardly ever analytical. In their time, these kinds of products served their purpose. With the advent of computer-driven animation systems, the potential of employing dynamic displays for analytical uses increased greatly. Tobler's (1970) population change in Detroit and Moellering's (1972; 1973a) 2-D example of traffic crashes, and his interactive and dynamic surface exploration (Moellering 1978), are examples of such analytical uses. Most animations regard time as a unitary vector of time, but Moellering showed with the traffic crash animation that one can treat time analytically, in a cyclical fashion, to bring out the important temporal fluctuations in the time sequence of the data. Many other research workers followed the path of dynamic cartographic visualization, as documented by Peterson (1995), MacEachren (1995), and MacEachren and Kraak (1997.

Spatial Data Standards

The development of spatial data standards involves a host of the conceptual topics addressed in the discussion preceding this section. Since the early 1980s, efforts have been undertaken to develop a unified set of spatial primitives, feature and attribute definitions, and a universal spatial data transfer mechanism that can transfer spatial databases between independent systems that can have different internal spatial data structures and computer architectures. In the United States, beginning in 1982 (Moellering 1982) this resulted in the development of the Spatial Data Transfer Standard (NCDCDS 1988), now known as Federal Information Processing Standard (FIPS) 173-1. Development of this spatial data standard involved many of the concepts discussed herein. During this time, many other countries and international organizations in the world succeeded in developing their own national or international spatial data transfer standards. (See Moellering and Hogan (1997) for details.)

In 1995, the International Standards Organization (ISO) Technical Committee 211 (TC211) on Geographic Information/Geomatics was founded in Oslo, Norway. Work is continuing in areas such as spatial primitives, spatial data schema, feature definitions, attribute schema, data quality, temporal data schema, data transfer, spatial metadata, imagery and gridded data. Real success of this work will mean that both the syntax and semantics of spatial data and related operators are successfully processed by such systems. The TC211 web site (ISO/TC211 1995) provides a wealth of information on spatial data standards activities that involve some of the best minds from the worlds of spatial data, GIS, analytical cartography, and information technology.

The holy grail for these spatial data standards efforts is the concept of interoperability, whereby consistent syntax and semantics would exist for both the spatial data and for the spatial operations that a conforming system could carry out. As Laurini (1998) points out, "...interoperability being a dream for users and a nightmare for systems developers... ." A full solution to this problem will take not only wide deployment of many of the analytical and theoretical concepts discussed here, but also the extension of these, and possibly the development of additional new concepts and theory in all of the fields mentioned above. For some individuals and organizations, this need is a major driving function of their professional activities.

Future Directions in Analytical Cartography

With all of the discussion that lays out much of the current research work going on in the field, the situation for continued conceptual and theoretical development may appear to be potentially confusing. Given that deep structure analysis in the field has only been a going concern for the last two decades there is much room for future development. One can envision continuing to extend and refine the use of spatial neighborhood operators, especially in the deep structure frequency domain. Not very many researchers have delved into this domain, and much new work in this area is indeed possible. There is also a great need to conduct much more work to extend the power of spatial neighborhood operators, in both the data domain and the frequency domain, and into the irregular cellular domain. Most of the spatial theory initially borrowed from other fields and adapted into analytical cartography existed in the regular square cellular domain. Now it is the responsibility of researchers in the field itself to extend this work into the irregular cellular domain. This could be a real challenge, when the cellular system is no longer geometrically or topologically isotropic but rather varies as to the size and number of cellular neighbors, and several other spatial properties.

One can also see that work will also continue on the problem to extend the work on spatial objects, especially those in the 3-D domain. Currently, 3-D entities can be modeled with combinations of 0-, 1-, and 2-D primitives from the national standard to form 3-D objects with higher dimensional properties. However, one would greatly like to have direct 3-D primitives available for spatial modeling. Currently, the only 3-D object primitive in the national transfer standard is that of a "voxel," the simplest "geometry-only" 3-D object. Research is going on to sort how to specify the 3-D objects that have properties of both geometry and topology. Work in related areas may provide additional suggestions. One can foresee such research continuing until the full complement of 3-D spatial objects with geometry and topology has been developed, and that one can envision them in some sort of "periodic table" of spatial objects. Data structures and databases are also of concern, and continued work on them is also assured, although, at present, more work in this area is probably going on in GISci and spatial database systems than in analytical cartography.

It also is not difficult to foresee continued work on temporal and spatial visualization. The question of how to characterize time in spatial data structures is a challenge, and will remain so for quite some time. Questions relating to the topology of time are only now being addressed; this will be a fruitful area of research for many years to come. Similarly, the new possibilities emerging in spatial analytical visualization offer additional avenues for interactive systems and spatial animations, and so this will continue, although cutting-edge research usually requires fairly sophisticated equipment. Reflecting back to Tobler's concept of the half-life of theory versus the half-life of technique, the questions to focus on in analytical cartography would probably be those that relate to the conceptual and theoretical advancement of the field over the technological advances. Other fields will address the technological advancements in this area.

Most of the research discussed in this review dealt with the theory and analysis of spatial data. A major need now is to sort out the conceptual approaches to spatial data semantics. If researchers, and many other people, are to be able to share spatial data seamlessly and successfully, then the research questions of spatial semantics and data interoperability must be solved. Although this problem may be seen as a technical problem there are many analytical and theoretical overtones to resolving this question.

Research Needs

Given that we are in the midst of a major innovation in the spatial data sciences, one might wonder just how many research opportunities are still available. A number, and they have been articulated by several researchers. Müller (1991), in his book on *Advances in Cartography* asked chapter authors to write a "projective summary" (or a statement of research needs) at the end of their chapters. Although this book has begun to age, a number of the research needs listed in the projective summaries are still very much valid because suitable analytical approaches to the problems have not yet been identified. Similarly, several of Moellering's (1994) research needs that have arisen during the Spatial Data Transfer Standard development effort remain unresolved, because they require better theoretical and conceptual understanding. Reporting on an NSF workshop on cross-disciplinary research and GIS, Mark (2000) identified the following research needs in GIScience:

- Examine the geospatial dimensions of software integration;
- Examine the scale and resolution aspects of spatial problems;
- Examine geographic aspects of process modeling; and
- Examine issues related to the usability of the human-computer interface.

The research agenda of emerging themes in GIScience research formulated by the University Consortium for Geographic Information Science was published in UCGIS (2000). Many of the sub-issues of that agenda flow out of the efforts described in this paper, and analytical cartography is in a good position to contribute to the solution of the analytical components.

Summary and Conclusions

Since its founding in the 1960s, analytical cartography has been developing as an important part of cartography and the related GIScience. The fundamental concepts of analytical cartography and their temporal aspects place the field in a good position to continue to make solid contributions to spatial science. The myriad of research topics offers s host of opportunities for continued research, as documented by the research needs articulated in this paper. What has sprung from Tobler's insights on analytical cartography in the early 1960s, has blossomed into a full-fledged research area. The future of analytical cartography, and the opportunities therein, appear bright indeed.

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REFERENCES

Abdelguerfi, Madhri, et.al. 1998 Representation of 3-D elevation in terrain databases using hierarchical triangulated irregular networks: A comparative analysis. *International Journal of Geographic Information Science* 12(8): 853-73.

Andrienko, G., and N. Andrienko. 1999. Interactive maps for visual data exploration?. *International Journal of Geographic Information Science* 13(4): 355-74.

Armstrong, M. 2000. Geography and computational science. *Annals of the Association of American Geographers* 90(1): 146-56.

Band, L. 1986. Topographic partitioning of watersheds with digital elevation models. *Water Resources Research* 22: 15-24.

Barber, C., R. Cromley, and R. Andrle. 1995. Evaluating alternative line simplification strategies for multiple representations of cartographic lines. *Cartography and Geographic Information Systems* 22(4: 276-90.

Bennett, D., and M. Armstrong. 1996. An inductive knowledge-based approach to terrain feature extraction. *Cartography and Geographic Information Systems* 23(1): 3-19.

Bergougnoux, P. 1997. Bridging the gap between computer science and geography. *GeoInformatica* 1(1): 5-9.

Brassel, K. 1974. A model for automated hill-shading. *American Cartographer* 1: 15-27.

Brassel, K., and R. Weibel. 1988. A review and conceptual framework of automated map generalization. *International Journal of Geographic Information Systems* 2(3): 229-44.

Brewer, C. 1997. Spectral schemes: Controversial color use on maps. *Cartography and Geographic Information Systems* 24(4): 203-20.

Brewer, C. (ed.). 1999. The State of U.S. Cartography: The 1999 U.S. National Report to the International Cartographic Association. *Cartography and Geographic Information Science* (26) 3: 165-236.

Brooks, F. 1999. What's real about virtual reality? *IEEE Computer Graphics and Applications* 19(6): 16-27.

Broome, F., and D. Meixler. 1990. The TIGER data base structure. *Cartography and Geographic Information Systems* 17(1): 39-47.

Brownrigg, D. 1984. The weighted median filter. *Communications of the ACM* 27 (8): 807-18.

Bunge, W. 1962. *Theoretical geography*. Lund Studies in Geography, Series C. Lund, Sweden: C.W.K. Gleerup.

Cartwright, W. 1999. Extending the map metaphor using Web delivered multimedia. *International Journal of Geographic Information Science* 13(4): 335-53.

Casetti, E. 1999. The evolution of scientific disciplines, mathematical modeling, and human geography. *Geographical Analysis* (31)4: 332-9.

Chen, Z-T. 1986. Spatial filtering of polygon data. In: *Proceedings, Second International Symposium on Spatial Data Handling*, Seattle. pp. 86-101.

Chomky, N. 1965. *Aspects of the theory of syntax*. Cambridge, Massachusetts: MIT Press.

Chrisman, N. 1972. Personal communication, statements made during the First International Symposium on Computer-Assisted Cartography, Reston, Virginia.

Chrisman, N. 1999. A transformational approach to GIS operations. *International Journal of Geographic Information Science* 13(7): 617-37.

Clarke, K. 1990. Map-based transformations (chap. 9); Map data structure transformations (chap. 10). In: *Analytical and computer cartography*, 1st ed. Englewood Cliffs, New Jersey: Prentice Hall. 334 p.

Clarke, K. 1995. *Analytical and computer cartography*, 2nd ed. Englewood Cliffs, New Jersey: Prentice Hall, 334 pp + software floppy disk.

Clarke, K. 2000. On the origins of analytical cartography. *Cartography and Geographic Information Science* (27) 2: this issue.

Clarke, K., and D. Schweitzer. 1991. Measuring the fractal dimension of natural surfaces using a robust fractal Estimator. *Cartography and Geographic Information Systems* 18(1): 45-52.

Clemintini, E., and P. Felice. 1997. A global framework for qualitative shape description. *GeoInformatica* 1: 11-27.

Corbett, J. 1979. *Topological principles in cartography*. Technical Paper 48. U.S. Bureau of the Census. 50 p.

Cressie, N. 1993. *Statistics for spatial data*, rev. ed. New York, New York: John Wiley & Sons.

deBerg, M., M. van Kreveld. and S. Schirra. 1998. Topologically correct subdivision simplification using bandwidth criterion. *Cartography and Geographic Information Systems* 25(4): 243-57.

Dettori, G., and E. Puppo. 1996. How generalization interacts with the topological and metric structure of maps. In: *Proceedings, 7th International Symposium on Spatial Data Handling*. Vol. II, pp. 9A.27 - 9A.38.

Dougherty, D., and H. Moellering. 1996. Using the 2-dimensional Fourier transform for numerical terrain analysis. In: *1996 ACSM Technical Papers, vol. 3: Proceedings: Surveying & Cartography*. pp. 268-77.

Douglas, E., and T. Peucker, T. 1973. Algorithms for the reduction of the number of points required to represent a digitised line or its caricature. *The Canadian Cartographer* 10(2): 112-23.

Duckham, M., and J. Drummond. 2000. Assessment of error in digital vector data using fractal geometry. *International Journal of Geographical Information Science* (14)1: 67-84.

Dutton, G. (ed.). 1978. *First International Advanced Study Symposium on Topological Data Structures for Geographic Information Systems, Proceedings*. Harvard Papers on Geographic Information Systems, Laboratory for Computer Graphics.

Dutton, G. 1981. Fractal enhancement of cartographic line detail. *The American Cartographer* 8: 23-40.

Dutton, G. 1989. Modeling locational uncertainty via hierarchical tessellation. In: Goodchild, M., and S. Gopal (eds.), *Accuracy of spatial databases*, New York, New York: Taylor and Francis. pp. 123- 40.

Dutton, G. 1999. Global hierarchical tessellations: Note 49. *Cartography and Geographic Information Science* 26(3): 224.

Eich, S. 2000. Visual discovery and analysis. *IEEE Transactions on Visualization and Computer Graphics* 6(1): 44-58.

Farrier, T., G. Tuell, and C. Heipke. 1995. Introduction to edge detection by wavelet analysis. In: Heipke (ed.) , *From digital images to spatial objects, Report No. 433*. Department of Geodetic Science and Surveying, Ohio State University. pp. 51-66.

Frank, A. 1991. Design of cartographic databases. In: Müller, J-C (ed.), *Advances in cartography*. London and New York, New York: Elsevier Applied Science. pp. 15-44.

Frank, A. 2000. Geographic information science: New methods and technology. *Journal of Geographical Systems* 2(1): 99-105.

Franklin, W. R. 2000. Applications of analytical cartography. *Cartography and Geographic Information Science* (27) 2: this issue.

Franklin, W. R., and M. Gousie. 1999. Terrain elevation data structure operations. In: *Proceedings, 19th International Cartographic Conference*, Ottawa, Canada. Vol 2, pp. 1011-20.

Getis, A. 2000. Spatial analysis and GIS: An introduction. *Journal of Geographical Systems* 2(1): 1-3.

Gold, C. 1999. The importance of the spatial model in analytical cartography. Paper presented to the 96th Annual Meeting of the Association of American Geographers, Honolulu, Hawaii.

Goodchild, M. 1979. *PLUSX documentation: Version 2.* Department of Geography, University of Western Ontario, London, Ontario.

Goodchild, M. 1980. Fractals and the accuracy of geographical measures. *Mathematical Geology* 12: 85-98.

Goodchild, M. 1990. Spatial information science: Keynote address. In: *Proceedings of the 4th International Symposium on Spatial Data Handling*, Zürich, Switzerland. Vol. 1, pp. 3-12.

Goodchild, M., and D. Mark. 1987. The fractal nature of geographical phenomena. *Annals of the Association of American Geographers* 77(2): 265-78.

Guptill, S. 1978. An optimal filter for maps showing nominal data. *Journal of Research of the U.S. Geological Survey* 6(2): 161-167.

Holloway, J. 1961. Smoothing and filtering of time series and space fields. In: *Advances in geophysics*. New York, New York, Academic Press. pp. 351-89.

Horn, B. 1982. Hillshading and the reflectance map. *Geo-Processing* 2: 65-146.

ISO/TC211. 1995. International Standards Organization Technical Committee 211 - Geographic Information/ Geomatics. See URL http://www.statkart.no/isotc211/.

Kimerling, A.J. 1989. Cartography. In: Gaile, G., and C. Willmott (eds.), *Geography in America*. Columbus, Ohio: Merrill Publishing Co. pp. 686-718.

Kirschenbauer, S., and M. Buchroithner. 1999. Real 3D Technologies for Relief Description. In: *Proceedings, 19th International Cartographic Conference*, Ottawa, Canada. Vol 2, pp. 975-81.

Kraak, M-J. 1994. Interactive modeling environment for three-dimensional maps: Functionality and interface issues. In: MacEachren, A., and F. Taylor (eds.), *Visualization in modern cartography*. Oxford, U.K.: Elsevier Science Ltd. pp. 269-85.

Kumler, M. 1994. An intensive comparison of Triangulated Irregular Networks (TINs) and Digital Elevation Models (DEMs). In: *Cartographica*. Monograph 45. 99 p.

Kunii, T., B. Falcidieno, and V. Savchenko (eds). 1997. *International Conference on Shape modeling and Applications*, Proceedings. Los Alamitos, USA: IEEE Computer Society Press.

Lam, N. 1983. Spatial data interpolation methods: A review. *American Cartographer* 10(2): 129-49.

Lam, N., and L. DeCola. 1993. *Fractals in geography*. Englewood Cliffs, New Jersey: PTR Prentice-Hall Inc. 308 p.

Laurini, R. 1998. Spatial multi-database topological continuity and indexing: A step towards seamless GIS data interoperability. *International Journal of Geographical Information Science* (12)4: 373 - 402.

Lin, F-T. 1998. Many sorted algebraic data models for GIS. *International Journal of Geographical Information Science* (12)8: 765-88.

MacEachren, A. 1995. *How maps work: Representation, visualization and design*. New York, New York: Guilford Press. 513 p.

MacEachren, A., and M. Kraak (eds.). 1997. Exploratory cartographic visualization: Special issue. *Computer & GeoSciences* 23(4): 335-494.

MacEachren, A., M. Wachowicz, R. Edsall, D. Haug, and R. Masters. 1999. Constructing knowledge from multivariate spatiotemporal data: Integrating geographical visualization with knowledge discovery in database methods. *International Journal of Geographical Information Science* (13)4: 311-334.

Maître, H., and J. Zinn-Justin (eds.) 1996. *Progress in picture processing – Les progrés du traitement des images.* Proceedings of the UJFG NATO Advanced Study Institute, 1992. Amsterdam, The Netherlands: Elsevier. 348 p.

Mandelbrot, B. 1967. How long is the coastline of Britain? *Science* 156: 636-8.

Mandelbrot, B. 1977. *Fractals: Form, chance, and dimension*. San Francisco, California: W.H. Freeman and Co. 365 p.

Marble, D. 1979. Integrating cartographic and geographic information systems education. *Proceeding, 39th Annual Meetings of ACSM,* Washington, D.C, 1979. pp. 493-99.

Marble, D. 2000. Some thoughts on the integration of spatial analysis and geographic information systems. *Journal of Geographical Systems* 2(1): 31-5.

Mark., D. 2000. Geographic information science: Critical issues in an emerging cross-disciplinary research domain. *Journal of the Urban and Regional Information Systems Association* 12(1): 45-54.

Marx, R., and F. Broome. 1985. A topologically based data structure for a computer readable map and geographic system. Paper presented to the 1985 AAG meetings, Detroit, Michigan.

McArthur, D., R. Fuentes, and V. Devarajan. 2000. Generation of hierarchical multiresolution terrain databases using wavelet filtering. *Photogrammetric Engineering & Remote Sensing* 66(3): 287-95.

McMaster, R., and S. Shea. 1992. *Generalization in digital cartography*. AAG Resource Publications for College Geography. Association of American Geographers, Washington, D.C. 134 p.

Miller, H. 1999. Potential contributions of spatial analysis to geographic information systems for transportation (GIS-T). *Geographical Analysis* 31(4): 373-99.

Miyashita, T., Y Kasahara, and T Tsurutari. 1985. A method of regional information overlay. *NEC Research & Development* 77: 22-30.

Moellering, H. 1972 *Traffic crashes in Washtenaw County, Michigan, 1968-1970*. A computer animated film. Highway Safety Research Institute, University of Michigan.

Moellering, H. 1973a. The computer animated film: A dynamic cartography. In: *Proceedings, Association for Computing Machinery*. pp. 64-9.

Moellering, H. 1973b. The computer animated film: A dynamic cartography. Paper presented to the 69th

Annual Meeting of the Association of American Geographers, Atlanta, Georgia.

Moellering, H. 1976. The potential uses of a computer animated film in the analysis of geographic patterns of traffic crashes. *Accident Analysis and Prevention* 8(4): 215-27.

Moellering, H. 1977a. Interactive cartographic design. In: *Proceedings of the American Congress on Surveying and Mapping*, 37th Spring meeting, Salt Lake City, Utah. pp. 516-30.

Moellering, H. 1977b. Real and virtual maps. Paper presented to the 73rd Annual Meeting of the Association of American Geographers, Salt Lake City, Utah.

Moellering, H. 1978. *A demonstration of the real time display of three-dimensional cartographic objects*. Computer animated videotape, Department of Geography, Ohio State University.

Moellering, H. 1980a. Strategies of real-time cartography. *Cartographic Journal* 17(1): 12-15.

Moellering, H. 1980b. The real time animation of three dimensional maps. *The American Cartographer* 7(1): 67-75.

Moellering, H. (ed.). 1982. *Papers from the Joint ACSM/ASPRS Session on Digital Cartographic Data Standards at the 1982 Meeting of the American Congress on Surveying and Mapping, Denver Colorado. Report #1: Issues in digital cartographic data standards.* National Committee for Digital Cartographic Data Standards, Columbus, Ohio.

Moellering, H. 1983. Designing interactive cartographic systems using the concepts of real and virtual maps. In: Wellar, B. (ed.), *Proceedings of the Sixth International Symposium on Automated Cartography*, Ottawa, Canada. Vol II., pp. 53-64.

Moellering, H. 1984. Real maps, virtual maps and interactive cartography. In: Gaile, G.L., and C.J. Willmott (eds.), *Spatial statistics and models*. Dordrecht, The Netherlands: Reidel Publishing. pp. 109-31.

Moellering, H. (ed.). 1987. *A draft proposed standard for digital cartographic data. Report #8: Issues in digital cartographic data standards.* National Committee for Digital Cartographic Data Standards, Columbus, Ohio.

Moellering, H. 1989a). A practical and efficient approach to the stereoscopic display and manipulation of cartographic objects. In: *Proceedings, Ninth International Symposium on Computer-Assisted Cartography*, Baltimore, Maryland, 1989. pp. 1-4.

Moellering, H. 1989b. An analytical approach to the cartographic display of satellite data in two and three dimensions. In: *Proceedings of the 14th ICA World Cartographic Conference*, Budapest, Hungary, 1989.

Moellering, H. 1990a. The interactive display of larger cartographic surfaces with shaded 3-D rendering. In: *Technical Papers of the 1990 ACSM/ASPRS Annual Convention, Vol. 2: Cartography*. ACSM/ASPRS. pp. 1-4.

Moellering, H. 1990b. 3-D stereoscopic surface exploration in interactive real time. In: *Proceedings of the 4th International Symposium on Spatial Data Handling*, Zürich, Switzerland. Vol. 2, pp. 381-9.

Moellering, H. (ed.). 1991a. *Analytical cartography*. Special content issue of *Cartography and Geographic Information Systems* 18(1).

Moellering, H. 1991b. Whither analytical cartography? *Cartography and Geographic Information Systems* 18(1): 7-9.

Moellering, H. 1991c. Research issues relating to the development of cartographic database transfer standards. In: Müller, J-C. (ed.), *Advances in cartography*. London, U.K.: Elsevier Applied Science. pp. 141-59.

Moellering, H. 1991d). Stereoscopic display and manipulation of larger 3-D cartographic objects. *Proceedings of the 15th International Cartographic Conference*, Bournmouth, England. pp. 122-9.

Moellering, H. 1993. MKS-ASPECT™ — A new way of rendering cartographic Z surfaces. In: *Proceedings of the 16th International Cartographic Conference*, Köln, Germany. Vol 1, pp. 675-81.

Moellering, H. 1994. Continuing research needs resulting from the SDTS development effort. *Cartography and Geographic Information Systems* 21(3): 180-9.

Moellering, H., H. Gauthier, and J. Osleeb. 1977. An interactive transit planning system based on individuals. *Journal of Urban Systems* 2: 93-103.

Moellering, H., and R. Hogan (eds.). 1997. *Spatial database transfer standards 2: Characteristics for assessing standards and full descriptions of the National and International Standards in the world.* London, U.K.: Pergamon/Elsevier Science. 369 p. + Crosstable.

Moellering, H., and J. Kimerling. 1990. A new digital slope-aspect display process. *Cartography and Geographic Information Systems* 17(2): 151-9. [Please note patents rights granted on this proprietary visualization process.]

Moellering, H., and J. Rayner. 1979. Shape analysis: Literature review. Typescript.

Moellering, H., and J. Rayner. 1980a. The harmonic measurement of shape in the complex domain. Paper presented to the 10th International Cartography Conference, Tokyo, Japan.

Moellering, H., and J. Rayner. 1980b. The harmonic analysis of spatial shapes using dual axis Fourier shape analysis. *Geographical Analysis* 13(1: 64-77.

Moellering, H., and J. Rayner. 1982. The dual axis Fourier shape analysis of closed cartographic forms. *The Cartographic Journal* 19(1): 53-9.

Moellering, H., and J. Rayner. 1984. The harmonic comparison of cartographic shapes using dual axis Fourier analysis (DAFSA). In: *Proceedings of the 12th International Cartographic Congress*, Perth, Australia. pp. 523-32.

Moellering, H., and W. Tobler. 1972. Geographical variances. *Geographical Analysis* 4: 34-50.

Molenaar, M. 1998. An introduction to the theory of spatial object modeling for GIS. London, U.K.: Taylor and Francis, 246 p.

Monmonier, M. 1989. Geographic brushing: Enhancing exploratory analysis of the scatterplot matrix. *Geographical Analysis* 21(1): 81-4.

Morrison, J. 1974. Philosophical-technical aspects of thematic cartography. *American Cartographer* 1(1): 5-14.

Muehrcke, P. 1973. *Thematic cartography*. AAG Monograph No. 19, Association of American Geographers, Washington, D.C.

Müller, J-C. (ed.). 1991. *Advances in cartography*. London and New York: Elsevier Applied Science.

Müller, J-C., J-P LaGrange, and R. Weibel. (eds). 1995a. *GIS and generalization*. London, U.K.: Taylor and Francis.

Müller, J-C., R. Weibel, J-P. LaGrange, and F. Salgé. 1995b. Generalization: State of the art and issues. In: Müller, J-C., J-P LaGrange, and R. Weibel, (eds), *GIS and generalization*. London, U.K.: Taylor and Francis.

Nadeau, D. 1999. Building virtual worlds with VRML. *IEEE Computer Graphics and Applications* 19(2): 18-29.

NCDCDS (National Committee for Digital Cartographic Data Standards). 1988. The proposed standard for digital cartographic data. *American Cartographer* 14(1): 142 pp + microfiche appendices.

Nyerges, T. 1980. Modelling the structure of cartographic information for query processing. Unpublished Ph.D. dissertation, Ohio State University, Columbus, Ohio. 203 pp.

Nyerges, T. 1981. Cartographic information modeling as a theoretical basis for cartographic data structures. Paper presented to the Second International HDBS Seminar, Richmond, Virginia, 1981.

Nyerges, T. 1991a. Analytical map use. *Cartography and Geographic Information Systems* 18(1): 11-22.

Nyerges, T. 1991b. Geographic information abstractions: Conceptual clarity for geographic modeling. *Environment and Planning A* 32: 1483-99.

Nystuen, J. 1963. Identification of some fundamental spatial concepts. *Papers of the Michigan Academy of Science: Arts and Letters* 48: 373-84.

Outcalt, S., K. Hinkel, and F. Nelson. 1994. Fractal physiography? *Geomorphology* 11: 91-106.

Palma, D., and R. Bendetti. 1998. A transformational view of spatial data analysis. *Geographical Systems* 5: 199-220

Parsons, J., and Y. Wand. 1997. Using objects for systems analysis. *Communications of the ACM* 40(12): 104-10.

Peterson, M. 1995. *Interactive and animated cartography*. Englewood Cliffs, : Prentice-Hall, New Jersey. 257 p.

Peucker, T. 1976. A theory of the cartographic line. In: *International Yearbook of Cartography*. pp. 134-42.

Peucker, T., and N. Chrisman. 1975. Cartographic data structures. *American Cartographer* 2(1): 55-69.

Peucker, T., and D. Douglas. 1975. Detection of surface-specific points by local parallel processing of discrete terrain elevation data. *Computer Graphics and Image Processing* 4: 375-87.

Peucker, T., R. Fowler, J. Little, and D. Mark. 1978. The Triangular Irregular Network. In: *Proceedings of the Digital Terrain Models (DTM) Symposium*, American Society of Photogrammetry. pp. 516-40.

Peuquet, D. 1984. A conceptual framework and comparison of spatial data models. *Cartographica* 21(4): 66-113.

Peuquet, D. 1999. Personal communication.

Quie. H-L. 1988. *Measuring the Louisiana coastline—An application of fractals*. AAG Cartography Specialty Group Occasional Paper #1, 33-40. Association of American Geographers, Washington, D.C.

Rayner, J. 1971. *An introduction to spectral analysis*. London, U.K.: Pion Ltd. 174 p.

Reichenbach, P., R. Pike, W. Acevedo, and R. Mark. 1993. A new landform map of Italy in computer-shaded relief. *Anno-Lii—Bollettion Di Geodesia e Scienze Affini* 1: 21 - 44.

Rhynsburger, D. 1972. Analytic delineation of Thiessen polygons. *Geographical Analysis* 5: 133-44.

Richardson, L. 1961. The problem of contiguity. *General Systems Yearbook* 6: 139-87.

Richardson, D. (ed.). 1996a. Spatial semantic, and temporal data integration for application in remote sensing and geographic information systems, Monograph 47. *Cartographica* 33(1): 105 p.

Richardson, D. 1996b. Automatic processes in database building and subsequent automatic abstractions. In: Richardson, D. (ed.), Spatial semantic, and temporal data integration for application in remote sensing and geographic information systems, Monograph 47. *Cartographica* 33(1): 41-54.

Richardson, D., and R. Thomson. 1996. Integrating thematic, geometric, and topologic information in the generalization of road networks. Monograph 47. *Cartographica* 33(1): 75-83.

Robinson, A. 1952. *The look of maps: An examination of cartographic design*. Madison, Wisconsin: University of Wisconsin Press. 105 p.

Robinson, A., and B. Petchenik. 1976. *The nature of maps: Essays toward understanding maps and mapping*. Chicago, Illinois: University of Chicago Press. 138 p.

Roman, G-C, and K. Cox. 1993. A taxonomy of program visualization systems. *IEEE Computer* 26(12): 11-24.

Rosenfeld, A, and A. Kak. 1982. *Digital picture processing*, 2nd ed. New York, New York: Academic Press.

Saalfeld, A. 1991. An application of algebraic topology to an overlay problem in analytical cartography. *Cartography and Geographic Information Systems* 18(1): 23-36.

Saalfeld, A. 2000. Complexity and intractability: Limitations to implementation in analytical cartography. *Cartography and Geographic Information Science* 27(2): this issue.

Salgé, F. 1991. Development of cartographic databases. In: J-C Müller, (ed.), *Advances in cartography*. London and New York: Elsevier Applied Science. pp. 45-70.

Shelberg, M., H. Moellering, and N. Lam. 1982. Calculating the fractal dimensions of empirical cartographic curves. In: *Proceedings of the Fifth International Symposium on Computer-Assisted Cartography (AUTO-CARTO 5)*, Crystal City, Virginia, 1982. pp. 481-490.

Terei, G. 1999. A thorough investigation of digital terrain model generalization using adaptive filtering. Unpublished Ph.D. dissertation, Department of Civil and Environmental Engineering, and Geodetic Science, Ohio State University. 151 p.

Thelin, G., and R. Pike. 1991. *Landforms of the conterminous United States—A digital shaded-relief portrayal*. U.S Geological Survey Miscellaneous Investigation Map, I-2206 + accompanying manuscript.

Tobler, W. 1961. Map transformations of geographic space. Unpublished Ph. D. dissertation, University of Washington, 183 p.

Tobler, W. 1965a. Computation of the correspondence of geographical patterns. *1964 Papers of the Regional Science Association* 15: 131-9.

Tobler, W. 1965b. Medieval distortions: The projections of ancient maps. *Annals, Assoc. of American Geographers* 56(2: 351-60.

Tobler, W. 1966., Personal communication via meetings of the Michigan Inter-University Community of Mathematical Geographers (MICMOG), and in his course in Analytical Cartography in the late 1960s.

Tobler, W. 1967. Of maps and matrices. *Journal of Regional Science* 7(2): 275-80.

Tobler, W. 1969a. The spectrum of U.S. 40. *Papers of the Regional Science Association* XXIII: 45-52.

Tobler, W. 1969b. Geographic filters and their inverses. *Geographic Analysis* 1(3): 234-53.

Tobler, W. 1970. A computer movie simulating urban growth in the Detroit region. *Economic Geography* 46: 234-240.

Tobler, W. 1973. A continuous transformation useful for districting. *Annals, New York Academy of Sciences* 119: 215-30.

Tobler, W. 1975. Linear operators applied to areal data. In: McCullagh, M. (ed.), *Display and analysis of spatial data*. pp. 14-37.

Tobler, W. 1976. Analytical cartography. *The American Cartographer* 3(1): 21-31.

Tobler, W. 1978. The comparison of plane forms. *Geographical Analysis* X(2): 154-72.

Tobler, W. 1979a. A transformational view of cartography. *American Cartographer* 6(2): 101-6.

Tobler, W. 1979b. Cartograms and cartosplines. In: *Proceedings, 1976 Workshop on Automated Cartography and Epidemiology*, Washington, D.C., Department of Health, Education & Welfare. pp. 53-8.

Tobler, W. 1979c. Smooth pynconphylactic interpolation for geographical regions. *Journal of the American Statistical Association* 74: 519 - 36.

Tobler, W. 1984. Application of image processing techniques to map processing. In: *Proceedings of the 1st International Symposium on Spatial Data Handling, Zürich, Switzerland*. pp. 140-144.

Tobler, W. 1986a. Pseudo-cartograms. *American Cartographer* 13(1): 43-50.

Tobler, W. 1986b. Measuring the similarity of map projections. *American Cartographer* 13(2): 135-9.

Tobler, W. 1994. Bidimensional regression. *Geographical Analysis* 26(2): 186-212.

Tobler, W. 1996. Converting administrative data on a sphere to a continuous surface. In: *Proceedings of the 3rd International Conference on Integrating Geographic Information Systems and Environmental Modeling*. CD-ROM.

Tobler, W. 2000. The development of analytical cartography: A personal note. *Cartography and Geographic Information Science* 27(2): this issue.

Toriwaki, J., and T. Fukumura. 1978. Extraction of structural information from grey pictures. *Computer Graphics and Image Processing* 7: 30-51.

USGIS (University Consortium for Geographic Information Science). 2000. Emerging themes in GIScience research. [UCGIS URL at: http://www.ucgis.org].

Van Roessel, J. 1991. A new approach to plane-sweep overlay: Topological structuring and line-segment classification. *Cartography and Geographic Information Systems* 18(1): 49-67.

Verbree, E., G. Van Maren, R. Germs, F. Jansen, and M-J. Kraak. 1999. Interaction in virtual world views—Linking 3D GIS with VR. *International Journal of Geographic Information Science* 13(4): 385-96.

Wagner, D. 1988. A method of evaluating polygon overlay algorithms. In: *Proceedings, ACSM-ASPRS Annual Convention*, St. Louis, Missouri, 1988. pp. 173-83.

Wang, D., et.al. 1983. Digital image enhancement: A survey. *Computer Vision, Graphics and Image Processing* 24: 363-81.

Warntz, W. 1966. The topology of a socio-economic terrain and spatial flows. *Papers of the Regional Science Association* 17: 47-61.

Wasilenko, M., and H. Moellering. 1977. *An information theoretic model for the derivation of choropleth classes*. Discussion Paper #56, Department of Geography, Ohio State University.

Weibel, R. 1992. Models and experiments for adaptive computer terrain generalization. *Cartography and Geographic Information Systems* 19(3): 133-153.

Weibel, R. (ed.). 1995. *Map generalization*. Special Content issue of *Cartography and Geographic Information Systems* 22(4).

White, D., J. Kimerling, and D. Overton. 1992. Components of a global sampling design for environmental monitoring. *Cartography and Geographic Information Systems* 19(1): 5-22.

White, D., J. Kimerling, K. Sahr, and L. Song. 1998. Comparing area and shape distortion on polyhedral-based recursive partitions of the sphere. *International Journal of Geographic Information Science* 12(8): 805 -27.

Wiechel, H. 1878. Theorie und Darstellung der Beluchtung von nicht gesetzmässig gebildeten Flächen mit Rücksicht auf die Bergzeichnung. *Civilingenieur* 24: 335-64.

Wilcox, D., and H. Moellering. 1995. Pass location to facilitate the direct extraction of Warntz networks from grid digital elevation models. In: *Proceedings of the 12th International Symposium on Computer-Assisted Cartography (AUTO-CARTO 12)*. ACSM Technical Papers. Vol. 4, pp. 22-31.

Williams, R. J. 1996. Cartography: A new era has begun. *Cartography* 25(1): 11-36.

Young, T., and K-S. Fu. 1986. *Handbook of pattern recognition and image processing*. New York, New York: Academic Press.