1 The satellite altimeter measurement

In the ideal case, a satellite altimeter measurement is equal to the instantaneous distance between the satellite’s geocenter and the ocean surface. However, an altimeter measurement is subject to many disturbances which have to be accounted for (Figure 1). An altimeter measurement may be written in the following form:

\[ h^* = h + h_{sg} + h_i + h_a + h_g + h_t + h_0 + \epsilon \]  

(1)

All the individual terms will be explained briefly, together with an accuracy assessment for the SEASAT and GEOSAT altimeter measurements. The left-hand side \( h^* \) represents the distance between the satellite’s geocenter and a chosen reference ellipsoid and is a result of a precise orbit computation. For example, using the JGM-3 gravity field model (complete to degree and order 70), the error of this term for GEOSAT is approximately 8 cm compared to a value for \( h^* \) of about 800 km. Because SEASAT is in a comparable orbit, the error of this term for SEASAT will be of the same order of magnitude. The actual measurement is represented by \( h \), with a value of about 800 km and a measurement noise \( \epsilon \) of 2-5 cm. The measurements have to be corrected for the position offset of the altimeter instrument from the satellite’s geocenter: \( h_{sg} \). This value can be determined very accurately before launch. Furthermore, corrections have to be applied to account for instrumental delays: \( h_i \). These corrections are on the order of a few decimeters with centimeter accuracy. The correction term \( h_a \) represents atmospheric path length delays, i.e. the altimeter measurement has to be reduced by this correction. This term can be divided into a tropospheric correction, on the order of 2.5 m with an error of 1-2 cm, and an ionospheric correction, on the order of 30 cm with an error of 1 cm. An additional instrument correction, \( h_s \), has to be applied to account for the interaction between the radar pulse and the sea surface. This correction is mostly a few percent of the so-called significant wave height (SWH). This SWH is on the order of a few meters, but in extreme circumstances this SWH may be as big as 20 m. The error of this correction is 1-2 % of the SWH, i.e. 1-40 cm. However, on the average the SWH will be below 4 m, thus the error a few cm at most.

The geoid height and the height induced by the solid Earth and ocean tides are represented by respectively \( h_g \) and \( h_t \). For the long-wavelength part of the geoid, i.e. for wavelengths above 4000 km, the error of recent gravity field models is smaller than 10 cm. However, the total geoid error (for all wavelengths) is of the order of 60 cm. The total geoid signal is of the order of 30 m. The errors in modeling of solid-Earth and ocean tides are typically a few cm, respectively, for the large ocean basins. Close to the coast, these errors are larger. Finally, \( h_0 \), represents the sea surface topography. This topography is the elevation of the surface of the ocean above the geoid caused by ocean dynamics. This ocean topography consists of a permanent and a variable part. The permanent part is defined as the dynamic sea surface topography, \( h_{SST} \). This topography has an rms of about 65 cm. The variable part of the ocean surface topography is known to have an rms of about 10 cm.

If all corrections have been applied and the reference models have been used, the result
is the so-called sea height residual. This sea height residual can be represented as:

\[ \Delta h = \Delta h_g - \Delta r + h_{SST} + \sigma \]  

(2)

The right-hand side of this equation consists of the geoid error \( \Delta h_g \), the radial (\( \approx \) height) orbit error \( \Delta r \), the dynamic sea surface topography \( h_{SST} \) and a term \( \sigma \) representing the errors in all corrections, sea surface variability and measurement noise.

### 1.1 Temporal and spatial resolution

Many altimeter satellites fly so-called repeat orbits. This means that the satellite follows exactly the same ground track (projection of the orbit on the earth’s surface) for each repeat period. For example, ERS-1 has flown for a certain time in a 3-day repeat orbit, completing 43 orbital revolutions, TOPEX flies in a 10-day repeat orbit, completing 127 orbital revolutions, and GEOSAT has flown for more than two years in a 17-day repeat orbit, completing 244 orbital revolutions during each repeat. The respective ground track patterns are displayed in Figure 2. It can be seen that for longer repeat periods, the ground track pattern becomes more dense, i.e. the spatial resolution is enhanced.

The advantage of flying repeat orbits is that along the ground track altimeter measurements are made at the same location one time during a repeat period. Thus, for example...
3-day ERS repeat orbit

10-day TOPEX repeat orbit

17-day GEOSAT repeat orbit

Figure 2. Groundtrack pattern of different altimeter satellites.
for TOPEX each 10 days a measurement can be made at the same location. Thus at a certain location, height phenomena can be observed with a temporal resolution equal to the repeat period. Opposite to spatial resolution, the temporal resolution becomes worse with longer repeat periods. Thus, a trade off has to be made between temporal and spatial resolution when selecting a certain repeat orbit.

1.2 Altimeter measurement differences

Considering all the different phenomena that affect an altimeter measurement, it will be obvious that it is hazardous to use the altimeter measurement in the original form (Equation 1) e.g. as tracking data in precise orbit determination. For example, errors in the geoid $\Delta h_g$ and the ocean topography $h_{SST}$ are completely correlated with the radial position of the satellite, i.e. these errors can be completely compensated by a radial orbit error, and in addition, errors in the modeling of the ocean topography can be compensated by a geoid error. However, a special type of tracking observation can be formed in which most of these errors are canceled. This measurement is defined as the satellite altimeter height crossover difference, in the following referred to as crossover difference. A crossover difference is formed by subtracting two altimeter measurements made at the same geographical location, one of these measurements being made when the satellite was in a descending (southbound) pass, and the other being made when the satellite was in an ascending. A residual crossover difference is formed by subtracting the respective sea height residuals at these locations. In this way, all permanent components, like the geoid and permanent ocean topography, are canceled and with them errors in modeling these components. It can be shown that a crossover difference is for the greater part a representation of radial orbit differences and a residual crossover difference of the difference in radial orbit errors along the descending and ascending passes. It must be mentioned that, by forming a (residual) crossover difference, part of the information of the radial orbit differences (errors) gets lost. This is the part of the orbit difference (error) that is equal both for the ascending and descending pass and is referred to as the geographically correlated orbit difference (error).

Besides crossover differences also so-called collinear track altimeter differences can be made. If an altimeter satellite flies a repeat orbit (and many do), the satellite will produce altimeter measurements of the same location after each repeat. By taking the difference between such measurements all constant phenomena disappear and variable phenomena are enhanced.

The techniques of forming crossover and collinear altimeter measurement differences are successfully applied in a number of applications.
2 Applications of satellite radar altimetry

Satellite altimetry offers the possibility to measure sea surface heights globally. In addition, some altimeters measure height above land and sea ice as well.

Applications of satellite radar altimetry include:

- Geophysics. Density differences in the Earth’s crust cause local differences in gravity. These affect the topography of the sea surface. The sea surface at rest is always perpendicular to the (local) gravity so a "mountain" in local gravity shows up as a "hill" in the sea surface (Figure 3). This "mountain" can be both a real subsurface seamount or island, or it may be a local increase in density in the Earth’s crust. An accurate determination of the constant part of the sea surface (as opposed to the time-dependent part, mostly due to oceanographic influences) is made by averaging as much data as possible from as many satellites as possible;

- Oceanography. Since currents are detectable as slopes in the sea surface (section 2.2), the world’s ocean currents can be detected and monitored. Small scale features are visible as well, like eddies, which are generated by the large scale currents (by the Gulfstream, for example). Altimeter data are also used for tide modeling;

- Precise orbit determination. Radar altimeter measurements also provide information about the motion of the altimeter satellite. Thus these measurements can be used as a special type of tracking data;

- Glaciology. Certain altimeters offer the possibility to monitor (changes) of the polar ice caps and sea ice.
2.1 Gravity field

The sea surface is predominantly a reflection of the mass distribution of the earth. In case of an sea surface at rest and no currents, this surface will be equal to an equipotential surface, the potential defined by the earth’s gravitational field. This surface is referred to as the geoid. The gravitational potential $W$ of the earth is defined as the gravity potential plus the potential of the centrifugal force caused by the rotation, $\omega_e$, of the earth. In spherical coordinates:

$$ W = \frac{\mu}{r} \left( 1 + \sum_{l=2}^{\infty} \sum_{m=0}^{l} \left( \frac{a_e}{r} \right)^l \left( \bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda \right) \bar{P}_{lm} (\sin \phi) \right) + \frac{1}{2} \omega_e^2 r^2 \cos^2 \phi $$

where $\mu$ is the gravity parameter of the earth, $a_e$ the mean equatorial radius, $\bar{P}_{lm}$ is the fully normalized Legendre polynomial of degree $l$ and order $m$, $\bar{C}_{lm}$ and $\bar{S}_{lm}$ denote the fully normalized gravity field harmonic coefficients. The radius $r$, latitude $\phi$ and longitude $\lambda$ denote the spherical coordinates.

The geoid is obtained by defining a constant value for the potential for which the equipotential surface fits best through the geometric shape of the earth. In practice, the geoid height is computed with respect to a reference ellipsoid, thus the potential of this reference ellipsoid, $W_{ref}$, has to be subtracted first from the total potential $W$. A good approximation for the geoid height relative to this ellipsoid is given by:

$$ h_g = \frac{W - W_{ref}}{\gamma} $$

where $\gamma$ is the norm of the gravity acceleration on the reference ellipsoid ($\approx 9.8 \text{ m/s}^2$).

A large data set of altimeter measurements has been collected covering a time period of more than a decade and these data can be used to generate a model for the stationary sea surface by averaging out variable phenomena. Such a model is displayed in Figure 4. Visible in the sea surface are trenches, troughs, etc. on the ocean bottom. Thus the sea surface is partly a reflection of the structure of the earth.

2.2 Oceanography

The stationary sea surface can be approximated for the larger part by the geoid, a signal with a magnitude of about 30 m (-110/90 meters minimum/maximum). However, part of the sea surface elevation is caused by currents, both stationary and variable. These currents cause elevations with a magnitude of about 70 cm (locally elevations may have magnitudes larger than a few meters). Especially the sea surface elevation caused by the stationary part of ocean currents and the geoid are difficult to separate. However, for the longer wavelengths, down to 4,000 km, high-quality gravity field models exist based on independent data. These gravity field models are used to model the long-wavelength
Figure 4. Mean sea surface model of the South Indian Ocean based on satellite radar altimeter measurements.

geoid and subtract this geoid from the altimeter derived mean sea surface. Figure 5 displays the sea surface elevation derived in this way.

A map as displayed in this Figure can be used to derive stationary ocean currents. The current direction follows from the gradient of the sea surface elevation. It can be shown that the currents "follow" lines of constant height. By following these lines of constant height, large currents can be distinguished close to Japan, the Kuroshio current, close to South-Africa, the Agulhas current close to South-Africa, the Gulfstream in the North Atlantic, and the Circumpolar current around the Antarctic.

A solution for the stationary ocean currents can be obtained by assuming that the Coriolis force due to the rotation of the earth is equal to pressure gradients due to sea surface height differences. In that case the following relations can be derived:

$$u = -\frac{\gamma}{2\omega_s \sin \phi} \frac{\partial h_{SST}}{\partial y}$$
$$v = \frac{\gamma}{2\omega_s \sin \phi} \frac{\partial h_{SST}}{\partial x}$$

(5)
where $u$ and $v$ are the current velocities in the East and North directions, respectively. The East and North directions are denoted by $x$ and $y$.

Radar altimeter measurements can also be used for studying the variability of the ocean surface. This can be done by e.g. collinear track or crossover analysis, i.e. comparing altimeter measurements taken at exactly the same location and removing the mean. Data of different satellites can be combined. An example of such an analysis is displayed in Figure 6. The results displayed in this Figure hold for a near 3-year period, May 1995 - March 1998. Clearly visible are high-variability regions, most of which are at the same locations as the strong stationary currents, such as the Kuroshio and Gulfstream areas (compare with Figure 5). Thus, regions with strong stationary currents are mostly also regions with high sea surface height variability.

### 2.3 Orbit determination

At first instance it is not desired to use satellite altimeter measurements as pseudo tracking measurements in precise orbit determination. This is because in principle the exact height of the sea surface is not known beforehand and the height of the satellite can only be known from a satellite altimeter measurement by adding the measurement to the sea surface height. The sea surface height is exactly what one wants to measure. This way, priorities are reversed. Measuring sea surface is the primary objective of satellite altimetry, not orbit determination. Moreover, sea surface height signals might alias into the orbit of the satellite. For example, if the sea surface height is modeled with a cer-
Figure 6. Variability of the sea surface based on ERS-2 single satellite crossovers (top) and on ERS-2/TOPEX dual-satellite crossovers (bottom)
tain error, this error can be compensated by an equivalent (height) error in the satellite location. This has to be prevented. For example, in Section 1 it was mentioned that the geoid error can easily reach a level of 60 cm. By using altimeter measurements directly in precise orbit determination, this geoid error might show up in the computed satellite orbit.

However, there is a simple way of using satellite altimeter measurements in an efficient way in precise orbit determination. An acceptable method is to use crossover differences (Section 1.2). By subtracting a measurement made on an ascending track (denoted by $\text{asc}$) from a measurement made on a descending track (denoted by $\text{des}$), all constant terms are eliminated. The so-called crossover difference residual becomes with Equation 2:

$$\Delta h_{\text{cross}} = \Delta r_{\text{asc}} - \Delta r_{\text{des}} - \sigma_{\text{asc}} + \sigma_{\text{des}}$$  \hspace{1cm} (6)

The geoid and stationary sea surface terms have disappeared by forming a crossover difference. One of the remaining terms is $\Delta r_{\text{asc}} - \Delta r_{\text{des}}$, which is a difference of radial orbit errors. For many altimeter satellites, this term is larger than the other remaining term $\sigma_{\text{asc}} - \sigma_{\text{des}}$, which predominantly reflects sea surface height variability. The exception is TOPEX/POSEIDON. For this satellite very high accuracy orbits can be computed. Therefore, for this satellite crossover differences are not used in the orbit determination. However, they can still be used to validate the accuracy of computed orbits. For satellites like GEOSAT and ERS-1/2, crossover differences are used in the precise orbit computation.