FITTING, PORTAYAL AND MAPPING FOR THE PRODUCTION OF 2^{nd} ORDER SURFACES PHOTOMOSAICS

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ABSTRACT

The Daphni Monastery is known worldwide for its famous Byzantine mosaics. After the earthquake of 1999, the monastery suffered serious damage and the Greek Ministry of Culture took action for the conservation of the monument. Within the frame of the requested tasks, the laboratory of photogrammetry was assigned the task of creating very large-scale (1:5) cartographic developments of the Domes.

Although hitherto encountered in literature, the creation of cartographic developments is not a standardized and automated process. In addition, the demanded large-scale products called for the development of a very innovative and rather complicated approach.

In this paper a complete approach is described in order to create cartographic developments of 2^{nd}-order surfaces. The proposed process was successfully applied in this case.

INTRODUCTION

The Daphni Monastery is considered to be one of the most important specimens of Byzantine art and architecture and it is known worldwide for its famous Byzantine mosaics. The monastery was built in the 11^{th} century and is situated in the southeastern part of Attica near Athens. After the earthquake of 1999, the monastery suffered serious damage and the Greek Ministry of Culture took action for the conservation of the monument. For that reason the Laboratory of Photogrammetry of NTUA was assigned with the thorough survey and recording of the monument. In particular, the project involved the creation of a variety of products such as: horizontal plans at five different levels (1:25, 1:50), 26 elevations (1:25, 1:50) – 6 exterior and 20 interior- both in photomosaics and line drawings, upper views (1:25, 1:50) both in photomosaics and line drawings, photomosaics (1:5) of all Mosaics on planar or developable surfaces photomosaics (1:5) of all Mosaics on non-developable surfaces and a data base (GIS) with detailed architectural information.

The problem of producing photomosaics of the various details on developable surfaces (i.e. cylinders, cones etc) has been dealt with elsewhere (Georgopoulos et al., 2001).

This paper mainly deals with the creation of large-scale developments of 2^{nd}-order surfaces. Although there has been significant research on this area, it mainly involved grayscale images or single image applications. The demand for large-scale products led to the proposed approach, which is able to incorporate a significant number of images.

The application involves:

- data collection for interior work
- the fitting of a mathematically defined surface (in this case a sphere) to a 3D point cloud
- the procedure followed for the definition of a system suitable for the projection processes
- the creation of an intermediary model for the one-to-one correspondence between the points of the surface of the object and the points of the model surface
- the procedure for the production of the cartographic developments

The process followed for the fitting of a mathematically defined surface was based on the relevant articles presented by Faber, (2000) and Theodoropoulou, (2000).

With respect to the creation of cartographic development, the application was based on the relevant articles presented by Theodoropoulou, (1999) and Miniutti, (2000).

However, the core of the process lies within the intermediary model, which is created with utilization of the DEM. This model is used during the projection process in order for the mosaicing of the numerous images of the object to be possible.

Another very significant aspect of the process is the procedure followed for the selection of a reference system suitable for the projection.

It should be noted that the whole process was designed and implemented in the MATLAB environment and it utilizes data that are always collected within the frame of such applications i.e. geodetically collected control point coordinates, digital images and the respective orientations, DEM data etc.

DATA COLLECTION

Considering the demand in large detail, and the high quality of the final products, the data collection methods were properly adjusted. All photographs were taken with a Hasselblad camera (c = 50mm) from a distance H_{max} = 1.2m so as to obtain photos of a scale approximately k = 1:25. The photo base chosen ranged between 20 – 30 cm, in order to have a satisfying overlap in all cases. Additionally, during the photographing, colour slides Tungsten (ASA 200) were used with artificial lighting provided by two Soft-boxes.

The scanning of the obtained photos has been done with a resolution of 600 dpi. Finally, for the determination of the photo orientations, the collection of the DEM data and the digital process of the final products (orthophotos, line-drawings etc), various commercial and non-commercial programs were used.

SURFACE FITTING

The first stage of the process is the fitting of a mathematically defined surface on the 3D point cloud. For this process it was decided to work only with the geodetically determined control points, considering them as more accurate than those of the DEM. Taking into account that map projections are going to be used, the desired model surface is that of a sphere or an ellipsoid. In this application, the most appropriate mathematical surface proved to be a sphere.

Generally, the surface of a sphere is defined as a set of points, which satisfy the following equation (1):

(1)

$$F(\mathbf{x},z) = (x-x_o)^2 + (y-y_o)^2 + (z-z_o)^2 + R^2 = 0$$

where R = the radius of the sphere
x_o, y_o, z_o = the position of the centre of the sphere
x, y, z = object coordinates in ground coordinate system
F(**x**, z) = the *algebraic distance* between the position of a point **x** and the surface **z**;

This very model is used for the least squares adjustment. In particular, the least squares method was applied. However, in order for this method to be successful, the vector of the approximate values of the unknowns has to be determined (Faber, 2000). The method presented in that article is based on the general equation of the second order surface (2). The specific method mainly involves the calculation of the parameters of a triaxial ellipsoid; but by adjusting the method properly, the results may as well be used in this case leading to a very satisfying outcome.

(2)

$$F(z,x) = \alpha_{11}x^2 + \alpha_{22}y^2 + \alpha_{33}z^2 + 2\alpha_{12}xy + 2\alpha_{13}xz + 2\alpha_{23}yz + \alpha_{14}x + \alpha_{24}y + \alpha_{34}z + \alpha_{44} = 0$$

where $[a_{11}, a_{22}, a_{33}, \ldots, a_{44}]$ = the equation coefficients
x, y, z = object coordinates in ground coordinate system

Following the proposed process and solving the generalized eigenvalue problem, the values of the coefficients are calculated using all of the available control points.

After the values of the coefficients have been calculated, the geometrically interpretable parameters of the desired surface may be derived. In addition to the author's suggestions for the determination of the position of the centre of the surface, another step is suggested in this contribution in order for the other parameters to be derived.

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------Formation of the matrix a ------

a = [a11 a12/2 a13/2; a12/2 a22 a23/2 ; a13/2 a23/2 a33]

------Solution of the eigensystem a (rotation matrix Q and the eigenvalues b)

[Q,b] = eig(a)

------Calculation of the angles f, w, k ----------

φ = asin(Q(1,3))
cosφ = cos(φ);
cosω = Q(3,3)/cosφ;
ω = acos(cosω)
cosκ = Q(1,1)/cosφ;
κ = acos(cosκ)

------Calculation of the axes of the ellipsoid ---
M1 = [X-xm Y-ym Z-zm];
M = Q'*M1';
M = M';

D = zeros(L,1);
for i = 1:L
   D(i) = M(i,:)*b*M(i,:)';
end
d = mean(D)
a1 = sqrt(d/b(1,1))
b1 = sqrt(d/b(2,2))
c1 = sqrt(d/b(3,3))
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In case a rotational ellipsoid is going to be used, the derived values, for the rotations and the axes, can be used after some proper adjustment e.g. after averaging of some values (This process has successfully been applied on simulation data of rotational ellipsoids).

For the case of the sphere surface the rotations are considered indifferent at this stage, the coordinates of the centre are used as approximate values and the approximate value of the radius is calculated by averaging the values a1, b1 and c1.

Having calculated the approximate values of the unknowns, a least squares adjustment can be applied. Considering the position of the centre of the sphere (x_o, y_o, z_o) and the radius (R) as unknown and treating the 3D space coordinates of the control points (x, y, z) as observations, a least squares adjustment has been applied. The adjustment process, apart from the values of the unknown parameters, gives the corresponding variations and the overall standard deviation of the adjustment, which may be considered as an index for the quality of the adjustment.

REFERENCE SYSTEM DEFINITION

Usually the object coordinates refer to a geodetic system, which has a random origin and orientation within the 3D space. Such a system is not always proper for projection purposes and this is the main reason for the definition of a new coordinate system.

This new system should have its origin placed at the centre of the model. In addition, the orientation of the system is very important, because this factor is critical for the way that the object of interest is represented on the projection plane. It is easily understood that remaining rotations can result in unreal and unwanted distortions in the final product.

The rotations to be implemented were calculated with respect to the position of the object of interest and a reference plane defined by the edge of the dome.

The rotations ω and φ are calculated in order for the plane defined by the X-Y axes of the new system to become parallel to the reference plane, whereas the κ rotation is calculated in order for the object of interest to be centred. In this way, the representation of the object on the projection plane will have no distortions caused by remaining rotations. However, this last stage of calculations is completely optional and it may be omitted.

The basic algorithm developed for the definition of the rotations is a least squares adjustment that is based on the equation of a plane (3).

(3)
$P = a\,x + b\,y - z + c = 0$

where a, b , c = constant coefficients
x, y, z = object coordinates in ground coordinate system

After these coefficients are determined by the least squares adjustment, they are geometrically interpreted. The following code illustrates how the geometrical interpretation of the coefficients can be achieved

-----*Geometric interpretation of the a, b, c coefficients*-----

$\omega_o = \mathrm{atan}(b)$
$\varphi_o = \mathrm{atan}((-a)/(b*\sin(\omega_o)+\cos(\omega_o)))$

After the angles ω_o and φ_o are calculated, the κ_o may also be calculated using only one point that is considered to lie somewhere close to the centre of the object. Knowing the point's coordinates on the geodetic reference system, the origin and the ω_o and φ_o angles, the coordinates of the selected point can be calculated in the new system. In this case, only the x and y coordinates are of interest, as these two values are used for the calculation of the κ_o angle that shall be implemented.

(4)

$\kappa_o = \mathrm{atan}(dx/dy)$

where dx, dy = the x and y coordinates of the selected point after implementing translation and rotation

In Figure (1), an attempt is made to show the importance of this process. In the first case, the picture is developed without taking into account the rotations between the object and the initial reference system. In the second case, all the calculations have been made and the result is a rather improved developed image. In this occasion, the derived values of the angles were about 0.5-3 degrees. However, these rather small values have brought a significant change to the result and thus should not be left out of the calculations.

Figure 1. The significance of the reference system selection in development creation. Up: Developed image created without implementing any rotations. Down: Developed image created with implementation of the calculated rotation angles.

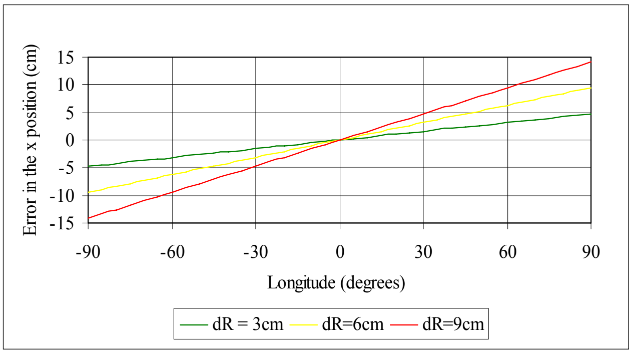

Figure 2. The way that the error in the radius of the model effects the x position on the projection plane.

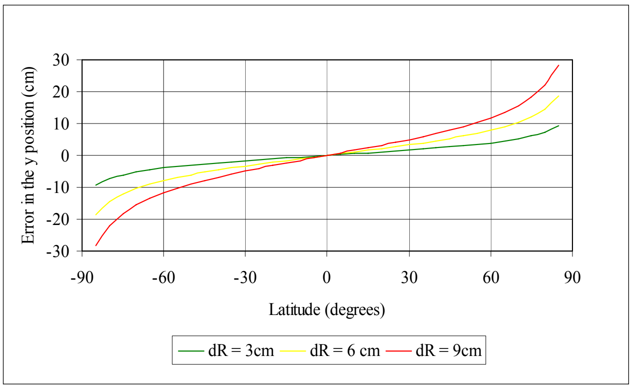

Figure 3. The way that the error in the radius of the model effects the y position on the projection plane.

THE INTERMEDIARY MODEL

When it comes to surface fitting, the most important problem is that the real surface is generally different from the model surface. As mentioned previously, an index for the quality of the adjustment is the variation of the distance between the points that are used for the adjustment and the mathematically defined surface. Assuming that the value of the variation (σ_o^2) is $0.001m^2$ and that the points follow the normal distribution, this means that 68% of the points are within a 3 cm distance from the surface, 95% of the points are within 6cm and 99% of the points are within 9cm. Furthermore, the error in the radius of the model indicates the very same thing. Taking this into account and assuming that a Mercator projection (Bugayevskiy & Snyder, 1995) is going to be used, the derivatives of the cartographic relationships with respect to the radius (R) of the model, which indicate the way that the error in the radius affects the x and y position of a point in the projection plane, are:

(5)

$$dx = \lambda\, dR$$
$$dy = \ln(\operatorname{atan}(\pi/4 + \varphi/2))\, dR$$

where dx, dy = the error in the position of a point on the projection plane
dR = the error in the radius
λ, φ = the longitude and latitude of a point

In the Figures (2) and (3), two graphs are presented to show the magnitude of the error in the x, y position of a point on the projection plane with respect to the values of the longitude and latitude. In each graph three series of data are presented in order for the reader to be able to estimate the magnitude of the error in the position, with respect to errors of different magnitude in the radius.

This fact obviously causes problems in mosaicing as the one-to-one correspondence between the points of the surface of the object and the points of the model surface cannot be ensured. This very problem is indicated in Figure (4).

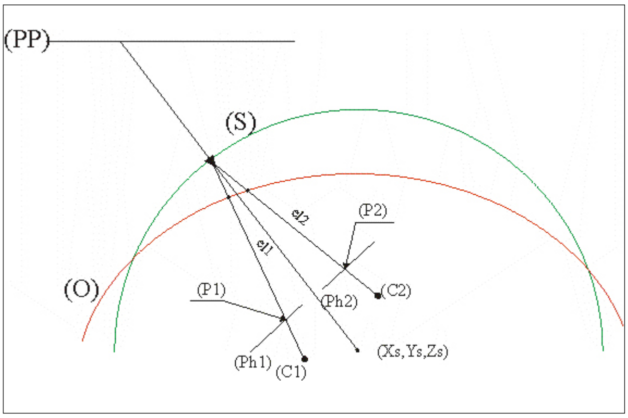

Figure 4. The problem caused due to the difference between the model and the actual object

In Figure (4), PP is the projection plane, O is the object surface, S is the surface of the sphere and X_S, Y_S, Z_S is the centre of the sphere. Beginning from a random position of the projection plane and applying the projection relationships, a point on the surface of the sphere is defined. As shown in Figure (4), the point that belongs on the sphere does not also belong on the surface of the object and when applying the collinearity equation for a stereo-pair, the images of two distinct points of the object surface are obtained.

In order for mosaicing to be possible, such problems should not appear. It is obvious that the problem could be solved if, instead of a point that belongs on the sphere, a point that belongs on the surface of the object could be obtained.

The solution to the problem is given by the creation of an intermediary model, which is based on the DEM data. This model helps to ensure the one-to-one correspondence between the points of the two surfaces and it is entirely based on directions.

After the new system is completely defined, the positions of the points of the DEM are expressed in this very system in spherical coordinates and the area of the object can be defined by the minimum and maximum longitude and latitude. This information is used for the creation of the intermediary model, which basically is a matrix. The breadth of the values in longitude and latitude is used for the determination of the size of the matrix. In this matrix the rows correspond to integer values of latitude whereas the columns correspond to integer values of longitude. Furthermore, in order to avoid trimming the edges of the object, the size of the matrix is increased. Each cell of the matrix contains the mean distance between the surface of the object and the centre of the model in the direction indicated by its position in the matrix. The construction of the matrix is done cell by cell. Beginning from the position of the cell (row, column), the corresponding values of latitude and longitude are calculated. The latitude and longitude are then used to detect all the points of the DEM within a search area 3 degrees wide. When the points of this area are detected, their mean distance from the centre is calculated and this value fills the corresponding cell. When no points of the DEM are detected in the search area, the value of the cell is set to the radius of the model. This way, a normalized model of the surface is obtained.

The whole idea might be very simple but it gave a very satisfactory solution to the problem of mosaicing.

CARTOGRAPHIC DEVELOPMENT CREATION

The next stage is the creation of the developed images. Given a specific cartographic projection and the DEM of an area of the object, the corresponding area of the developed image on the projection plane can be defined. For each position (x, y) the colour has to be obtained; using the inverse cartographic relationship, the corresponding latitude and longitude values are calculated. Instead of using the radius of the model, the distance between the surface of the object and the centre of the model is used. This parameter is found by interpolation on the intermediary model. This way, the full spherical coordinates of the position are obtained and then expressed in the reference system of the model in cartesian coordinates. Finally, the position is expressed in the initial geodetic system, and using the collinearity condition, the corresponding position on the photographic plane is determined; using the parameters of the interior orientation, the position on the digital image is found and the colour is obtained by interpolation.

The equations that describe the relationship between a position on the matrix of the intermediary model and the corresponding direction in the 3D space are:

(4)

$$m = f - \mathrm{fix}(minF) + 1$$
$$n = l - \mathrm{fix}(minL) + 1$$

where m, n = the size of the matrix of the intermediary model (rows, columns)
l, f = the parameters of latitude and longitude for a given point
$minL$, $minF$ = the minimum values of latitude and longitude respectively

After all of the developed images are created, the coordinates of the control points on the projection plane are also calculated. The outcome is an ASCII file, which is first converted to DXF format and then to an image, which is used for the process of mosaicing. In Figure (5), an example is presented. The mosaic illustrated consists of nine images that were created in the way described.

Figure 5. Cartographic development mosaic consisting of nine images.

Another issue is the research that must be done prior to the implementation. This process involves the selection of the most appropriate projection type and the way in which it shall be implemented so as to ensure that the product will be useful, reliable and suitable for the application it is designed for. In order for this to be possible, some kind of optimization has to take place so that the final products have the desired attributes and, most importantly, the least possible distortions. Another reason that makes this process obligatory is that the method is very time-consuming for projects that involve a large number of models. For example, in this case the mosaics produced would be used for preservation purposes. Taking this fact into account, it was chosen to work mainly with conformal projections. In particular, the Mercator Conformal Projection was implemented for the four pediments that adorn the nave of the Daphni Monastery. In each case, the implementation of the projection was different depending on the position of the main theme on the object surface. Additionally, a mosaic was created for the dome of the church, but this time using the Stereographic projection and taking into account the position and the area of the dome that was covered by the mosaic.

DISCUSSION

The proposed method had very satisfactory results even for surfaces that could not be very well defined. It is obvious that on cartographic developments no measurements can be made and thus the accuracy of such a product is of no importance. However, in cases where numerous models are to be incorporated, all the parts of the development should be accurately constructed in order to be coincident in overlapping areas and thus proper for mosaicing.

By implementing the proposed method, the results were satisfactory even for mosaics that consisted of a very large number of images (the largest mosaic consisted of 42 images).

Nevertheless, the method has some disadvantages. As mentioned at the beginning, the application was designed and implemented in the MATLAB environment. Apart from the advantages and the numerous possibilities that MATLAB offers, some very significant problems were encountered. The most important problem is due to the relatively small speed that can be achieved for the creation of very large images e.g. an image of approximately 40MB, can take about a quarter of an hour. Another problem is that there is some kind of restrain on the size of the images that can be produced e.g. it was impossible to create an image of the size of 80 MB. As proved through experimentation, these magnitudes and the respective limitations depend on the platform and the resources of the system used.

At this stage the method is rather time-consuming and could not be characterized productive. It is clear that the creation of an application that would be independent from MATLAB would be much faster and would not have such restrictions.

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